Rayleigh-Bénard Convection in a Vertically Oscillated Fluid Layer

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We report the first quantitative observations of convection in a fluid layer driven by both heating from below and vertical sinusoidal oscillation. Just above onset, convection patterns are modulated either harmonically or subharmonically to the drive frequency. Single-frequency patterns exhibit nearly solid-body rotations with harmonic and subharmonic states always rotating in opposite directions. Flows with both harmonic and subharmonic responses are found near a codimension-two point, yielding novel coexisting patterns with symmetries not found in either single-frequency states. Predictions from linear stability analysis of the onset Rayleigh and wave numbers compare well with experiment, and phase boundaries for coexisting patterns track single-frequency marginal stability curves.

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Characterizing pattern formation is a fundamental problem in the study of nonequilibrium systems. Wave number selection mechanisms provide one useful means for identifying common pattern forming behaviors in diverse physical systems [1]. The pattern wave number \( q \) may be selected by geometrical constraints; a canonical example of geometry-induced patterns is found in Rayleigh-Bénard convection where the pattern length scale is governed by the fluid layer thickness \( d \) [2]. By contrast, the selected \( q \) may depend on an externally imposed frequency \( \omega \) in systems subjected to spatially uniform, time-periodic oscillation [1]; a common example of these dispersion-induced patterns is the parametric excitation of surface waves (Faraday waves) in an open container of fluid [2]. Pattern selection by these generic mechanisms also arises in nonhydrodynamic systems; geometry-induced patterns occur in the buckling instability of thin plates [3], while dispersion-induced patterns are generated by optical waves in a fiber laser [4] and crystallization waves in \(^4\)He [5].

We report the first experimental observations of both geometry-induced (onset \( q \) weakly dependent on \( \omega \)) and dispersion-induced (onset \( q \) strongly dependent on \( \omega \)) patterns in a single system: a fluid layer that is both heated from below and vertically oscillated sinusoidally. Fluid motion in this system requires a thermally induced heated from below and vertically oscillated sinusoidally. Patterns exhibit nearly solid-body rotation over a wide parameter range with H and S patterns always rotating in opposite directions. In addition, we find and characterize a region of parameter space where the distinct spatial and temporal scales of H and S patterns interweave to form complex states [Figs. 1(e) and 1(f)], including localized domains of one pattern embedded in the other, mode-locking, and formation of pattern symmetries not found in either pure state.

Experiments are performed on a layer of CO\(_2\) gas bounded below by a 0.6-cm-thick gold-coated aluminum mirror, laterally by a 3.80 ± 0.03 cm inner diameter ring of filter paper, and above by a 2.54-cm-thick sapphire window. Two cell depths are studied: \( d = (6.50 \pm 0.03) \times 10^{-2} \text{ cm, corresponding to a vertical diffusion time of } \tau_v \approx d^2/\kappa \approx 2 \text{ s.} \) Length is scaled by \( d \) and time by \( \tau_v \). Thermal gradients are imposed across the fluid layer by heating the mirror from below and using circulating water to cool the window from above resulting in a vertical temperature difference (\( \Delta T \)) controlled to within ±0.01 °C. The fluid layer is vertically vibrated sinusoidally by a hydraulic piston under closed-loop control rendering oscillations with less than 4% of the total amplitude in higher harmonics. Patterns are visualized using shadowgraphy and recorded by a digital image acquisition system. To determine H or S amplitude modulation pattern images are captured at ~20 Hz (twice the drive frequency) while long-time dynamics are recorded at ~0.5 Hz using a shutter synchronized with the piston motion. For \( \delta = 0 \) (no oscillations), the conductive state loses stability to roll patterns, suggesting that non-Boussinesq effects are weak and occur below the limit of our temperature resolution. These observations are consistent with our calculations using a variational model described by previous authors [13–15], which demonstrate rolls are the globally stable state for \( R \) only ~0.3% larger than the unmodulated critical value, \( R_v^0 = 1708 \). Patterns are explored with...
H convection occurs for small $\delta$ [Fig. 2(a)]. Without oscillations ($\delta = 0$) spiral defect chaos arises for $R \geq 2500$ in agreement with previous experiments [6]. With oscillations ($\delta > 0$ at fixed $R$), spiral defect chaos modulated at $\omega$ persists for a significant range in $\delta$ (e.g., $\delta \lessapprox 3.30 \times 10^{-4}$ at $R = 4840$). With increasing $\delta$ the number of spiral defects decreases as more regular states whose morphology depends on $R$ emerge. For $2500 \lessapprox R \lessapprox 3900$ these emerging patterns are typically multiarm spirals which reduce in arm number, eventually becoming targets as the convection state is approached. At larger $R$ ($3900 \lessapprox R \lessapprox 5500$) spiral defect chaos becomes a pattern of nearly parallel rolls tending to terminate perpendicular to the sidewalls and possessing several foci at the boundaries; the number of foci and curvature of the associated rolls decreases with increasing $\delta$. The transition with increasing $\delta$ from spiral defect chaos to parallel rolls is reminiscent of the well-studied transition in unmodulated Rayleigh-Bénard convection for decreasing $R$ [16]. For $3100 \lessapprox R < 4560$ uniform parallel rolls or targets lose stability with increasing $\delta$ as domains of hexagons form [Fig. 1(b)]. These states of hexagons and rolls or targets occur only for a narrow range ($\approx 6 \times 10^{-6}$) of $\delta$ before losing stability to conduction with a small additional increase in $\delta$. Within the experimental resolution in $\delta$ ($\approx 2 \times 10^{-5}$) no hysteresis is observed in the transition between the hexagon-roll states and conduction. The nonhysteretic transition and morphology of these patterns are consistent with other modulated Rayleigh-Bénard experiments involving time-periodic driving of the bottom plate temperature [17].

S convection is observed for sufficiently large $\delta$ [Fig. 2(a)]. The onset of S patterns occurs as a uniform patch of rolls; no hysteresis or hexagons are observed, consistent with the S temporal symmetry that excludes three wave interactions [2]. With increasing $\delta$ other roll domains form with grain boundaries at the domain intersections. The roll domains merge with further increase in $\delta$, leading to the formation of disclinations that may interact [Fig. 1(c)]. For sufficiently large $\delta$, either a single convex disclination or, less frequently, a spiral arises centered within the convection cell. These patterns experience skew-varicose instabilities leading to repeated

FIG. 1. Convection patterns are visualized using shadowgraphy and characterized by four dimensionless quantities: Prandtl number $Pr = \frac{\nu}{\kappa}$, driving frequency $\omega = \frac{2\pi}{T}$, displacement amplitude $\delta = \frac{x^2}{\pi \nu}$, and Rayleigh number $R = \frac{\nu g \Delta \rho}{\kappa}$. (a) H spiral defect chaos ($\delta = 1.76 \times 10^{-4}$, $R = 3198$). (b) Coexisting H rolls and hexagons ($\delta = 3.74 \times 10^{-4}$, $R = 4216$). (c) S rolls near onset ($\delta = 4.26 \times 10^{-4}$, $R = 3958$). (d) S rolls ($\delta = 4.05 \times 10^{-4}$, $R = 4990$). (e) H rolls with localized domains of S rolls ($\delta = 3.76 \times 10^{-4}$, $R = 4962$). (f) S rolls containing grain boundaries overlaying a weak pattern of H rolls and cells ($\delta = 3.64 \times 10^{-4}$, $R = 5424$).

$\omega$ and $Pr$ held constant (Fig. 1) while increasing and decreasing $\delta$ at various fixed values of $R$.
nucleation of dislocations; additionally the patterns may move off center [Fig. 3(b)]. With increasing $\delta$ a single roll domain forms with few dislocations and a long wavelength distortion [Fig. 1(d)]. Patterns qualitatively similar to Fig. 1(d) have been previously observed in rotating Rayleigh-Bénard convection [18].

Following the method described by previous investigators [10,11] we performed a linear stability analysis for both critical Rayleigh numbers $R_c$ and critical wave numbers $q_c$ are in good agreement with the experimentally observed values at onset of both H and S convection (Fig. 2). For H convection, modulation enhances the stability of conduction ($R_c > R_0^H$) while decreasing $q_c$, below its unmodulated value $q_0^H = 3.117$, consistent with previous modulated Rayleigh-Bénard experiments [17]. In addition, for S convection $R_c > R_0^S$ and $q_c$ decrease with increasing $\delta$ (Fig. 2). For parameter values not studied here $R_c$ is predicted to drop below $R_0^H$ [10].

For $R \geq 2500$ patterns undergo nearly solid-body rotation where H and S states rotate opposite directions (Fig. 3). For fixed $R$ ($2500 \leq R \leq 4560$) and increasing $\delta$ from zero, the onset of rotation occurs near $\delta = 2 \times 10^{-4}$. Patterns deviate somewhat from ideal solid-body rotation because point defects and grain boundaries continually propagate within the rotating patterns. Global rotation rate increases with $\delta$ except near the conduction boundaries where rotation slows as patterns weaken [Fig. 3(c)]. A given rotation direction is selected and maintained by the patterns throughout the duration of an experimental trial. Patterns do not equally select clockwise and counterclockwise directions; in 62 separate experiments H states rotated counterclockwise in 84% of the trials. In all cases, H and S patterns rotate in opposite directions. Rotations are qualitatively robust against perturbations from tilting the apparatus $\sim 5^\circ$ off the vertical, changing the sidewalls to square symmetry and asymmetric cooling of the top plate.

For $R > 4560$ conduction is no longer stable; instead H and S patterns compete and coexist over a range of $\delta$ between the pure states [Fig. 2(a)]. As $\delta$ is increased, pure H states lose stability to mixed patterns where localized patches of S rolls form about H defects and are advected along as the defects propagate. At slightly larger $\delta$ [e.g., $\delta = 3.67 \times 10^{-4}$ in Fig. 4(e)], S rolls begin to form perpendicular to H upflows throughout the pattern [Fig. 1(e)].

The wave number of emerging S rolls ($q_S$) is close to the second harmonic of the H pattern wave number ($q_H$). A small change in $\delta$ [e.g., $\delta = 3.69 \times 10^{-4}$ in Fig. 4(e)] yields states where H patterns of local hexagonal, square, and rhombic symmetries are mixed with rolls of the S component perpendicular to the cell faces [Figs. 4(a), 4(c), and 4(d)]. For these states, the H and S components contribute equal power to the wave number spectra and have mode-locked wave numbers ($\frac{q_S}{q_H} = 3$). With further
small increases in $\delta$ [$\delta \approx 3.72 \times 10^{-4}$ in Fig. 4(e)], the S component dominates the power spectra and, concurrently, the wave number ratio unlocks ($q_H^{2.8} < 2.8$ as $q_H$ increases abruptly. The S component forms domains of increasingly larger size as the H component gradually weakens [Fig. 1(f)]. Upon crossing the phase boundary with purely S states [Fig. 2(a)] rolls with a long-wavelength distortion are typically observed [Fig. 1(d)].

The experimentally determined phase boundaries separating coexisting states from the pure patterns track the marginal stability curves for the conduction state [Fig. 2(a)]. For $R > 4560$, the H marginal stability curve is in nearly exact agreement with the phase boundary between coexisting and pure S states. This suggests the S base state from which H convection bifurcates differs little from conduction in a spatially averaged sense. Spatial Fourier spectra support this viewpoint since the higher modes of S patterns cannot overlap with the smaller wave number H fundamental. By contrast, the experimentally determined phase boundary between coexisting and pure H states lies above the S marginal stability curve, suggesting that H convection inhibits the onset of S convection due to wave number interaction. Evidence for this inhibitory effect is further bolstered by the observation that S convection first appears near H pattern defects. The amplitude of convective flow is generally suppressed in the cores of pattern defects [19] and, therefore, any inhibitory effect of H convection on S convection is delayed by the presence of H convection [10].

These multiple length scale convection patterns differ qualitatively from coexisting wavelength states in spatially separate domains observed in optical systems [20] as well as quasiperiodic [21] and superlattice [22] states reported in Faraday experiments. Three wave interactions (resonant triads) are responsible for multiscale Faraday patterns; it seems doubtful resonant triads are important in the convection patterns described here due to the S temporal symmetry and large difference between $q_H$ and $q_S$. Resonant triads may be introduced in convection patterns by non-Boussinesq effects and for the current experiment with heating from above squares and quasiperiodic structures have been predicted [12].

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