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Observation of the Wannier-Stark fan and the fractional ladder in an accelerating optical lattice

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We report an experimental study of the Wannier-Stark fan and the fractional Wannier-Stark ladder using laser-cooled sodium atoms in an accelerating one-dimensional standing wave of light. We prepare the atoms in the lowest motional band of the optical lattice and then impose a constant acceleration. A weak oscillatory component is added to the acceleration in order to resonantly drive interband transitions, and the number of atoms that remain in the lowest band is measured as a function of the probe frequency. The spectrum is characterized by a ladder of resonances spaced by the atomic Bloch oscillation frequency ω_B. When an additional, strong ac component at frequency Ω is added, a fractional ladder is observed with a spacing related to the electric matching ratio Ω/ω_B. [S1050-2947(99)51209-2]

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While the nature of the eigenstates of a stationary, periodic potential was pointed out almost 70 years ago by Felix Bloch, the behavior of these states when static or time-dependent fields are added has remained until the present day a subject of intense investigation. One of the most fundamental predictions made by Bloch and later proved by Zener and Jones was that a particle in a periodic potential will oscillate rather than accelerate in response to a homogeneous static field [1]. It was argued by Wannier that this phenomenon, known as Bloch oscillations, would be associated with the formation of a discrete ladder of energy states, the Wannier-Stark (WS) ladder, spaced by the Bloch oscillation frequency [2]. These predictions were the subject of debate until this decade, when these effects were observed in experiments using superlattices and then optical lattices [3,4].

Advances in the field of laser cooling have provided a new testing ground for quantum transport, that of atomic motion in optical lattices. Because of their perfect spatial periodicity and almost complete lack of dissipative effects, optical lattices provide a clean system for the study of coherent quantum transport phenomena. A previous experiment presented by our group spectroscopically studied the atomic WS ladder of atoms in an accelerating optical lattice [5]. This work concentrated on the observation of the Bloch frequency. Recent improvements in our experiment have now enabled the measurement of the absolute location of the WS resonances as the acceleration is scanned over a wider range, resolving the complete WS fan. In addition, by introducing a strong ac field of a particular frequency, the WS ladder is observed to splinter into the fractional ladder, an effect predicted to occur in lattices but one that has never, to our knowledge, been observed to date [6].

When a periodic potential of period d is imposed on a free particle, the hitherto continuous energy versus momentum dispersion relation is broken up into bands of allowed energies separated by band gaps. Both the eigenstates and eigenenergies $E_n(k)$ are periodic functions of k with period $2\pi/d$, where n is a discrete index labeling the band and k is the continuous quasimomentum of the state. For this reason, k is conventionally restricted to lie in the first Brillouin zone $(-\pi/d,\pi/d]$. As a consequence of the energy degeneracy of the potential wells, a wave packet initially localized in one well will spread via resonant *Bloch tunneling*, a phenomenon analogous to the spreading of a Gaussian wave packet in free space.

When a dc field is applied, producing a constant force F on the particles, the quasimomentum evolves like a free particle momentum $\hbar\dot{k}=F$ sweeping repeatedly through the first Brillouin zone. In addition to changing the quasimomentum, the force can induce interband Landau-Zener (LZ) transitions changing the band index by one with the highest probability where the band gap is smallest. If the force is sufficiently weak, however, the band index will remain unchanged and a particle will return to its initial state (up to a phase factor) every Bloch period, $T_{\mathrm{B}}=h/Fd$. Since the eigenenergies are periodic functions of k, the velocity of the particle, given by $v=(1/\hbar)[\partial E_n(k)/\partial k]$, will show the same periodic behavior, the Bloch oscillation. In addition, because the translational symmetry is broken by the applied force, Bloch tunneling stops, and the states become localized in position. The resulting quasi-bound-states, which constitute the WS ladder, are metastable with a lifetime determined by the LZ tunneling probability and are spaced by an energy $E_B=h/T_B=\hbar\omega_B$ proportional to the Bloch frequency. When an ac field of frequency Ω is added in addition to the dc field, the WS ladder states are predicted to further split into a fractional ladder with a new spacing, depending on the value of the electronic matching ratio Ω/ω_B [6].

Our experimental system consists of laser-cooled sodium atoms in a near-resonant, far-detuned standing wave of light. For sufficiently large detuning from resonance, the effect of spontaneous emission can be neglected and the excited-state amplitude can be adiabatically eliminated from the Schrödinger equation, resulting in an effective one-dimensional potential for an atom in its internal ground state given by $V_0\cos(2k_Lx)$, where $k_L=2\pi/\lambda_L$ is the photon wave number. The amplitude of this optical dipole potential V_0 is proportional to the laser intensity and inversely proportional to the detuning from the atomic resonance.

In order to realize the external homogeneous dc and ac fields, we accelerate and modulate respectively, the position of the optical potential by ramping and modulating the frequency difference of the two counterpropagating beams forming the standing wave. In the stationary frame, the potential has the form $V_0\cos[2k_Lx-\phi(t)]$ where $\phi(t)=\int_0^t\Delta\omega(t')dt'$ is the total accumulated phase difference of the two beams with instantaneous frequency difference $\Delta\omega(t)$. We choose $\Delta\omega(t)=2k_Lat-\delta\sin(\Omega t)$, where a is the acceleration, Ω is the modulation frequency, and δ is the frequency excursion of the frequency modulation (in rad/s). If we make a unitary coordinate transformation into the reference frame in which the potential is stationary, the Hamiltonian has the form

$$H=\frac{p^2}{2M}+V_0\cos(2k_Lx)+aMx+Mx\frac{\delta\Omega}{2k_L}\cos(\Omega t), \quad (1)$$

where the mass of the atom M appears in the ac and dc forcing terms, revealing their inertial origin. For our system, the atomic Bloch frequency is $\omega_B=2\pi Ma/2\hbar k_L$.

In order to observe the spectrum of Eq. (1), we add an additional frequency modulation at ω_p to the standing wave, which is much weaker than the strong ac field. This probe modulation does not significantly modify the spectral features but can drive transitions between appropriate states when on resonance. By preparing the atoms in the first band and measuring the depletion of its population as a function of the probe frequency, a spectrum was obtained.

Our experimental setup for the spectroscopic study of an optical lattice is based on the system previously used to study Rabi oscillations between Bloch bands and dynamical band suppression [7]. The optical standing wave was created by two linearly polarized, counterpropagating beams from a single dye laser. The frequency difference of the two beams was controlled by an acousto-optic modulator in a double-passed configuration to minimize beam deflection. The beams were spatially filtered to have a Gaussian profile with a waist of 2.0 mm at the location of the atomic cloud. The average power in each beam was in the range from 80 to 100 mW and was monitored with two calibrated photodiodes during the experiment. The light was detuned far from the $(3S_{1/2},F=2)\leftrightarrow(3P_{3/2},F=3)$ transition at 589 nm, and the detuning ranged from 30 to 40 GHz.

Several steps were necessary to prepare a significant number of atoms in the lowest band of the optical potential. First, approximately 10^5 sodium atoms were collected and cooled in a vapor cell, magneto-optic trap in a σ^+-σ^- configuration (MOT) [8]. After the cooling and trapping stage, the MOT fields were extinguished and the standing wave was turned on. Approximately 10% of our atoms were projected into the lowest band of the potential. We then accelerated the potential at 2000 m/s^2 for 600 μs. This acceleration was chosen to maximize the tunneling out of the second and higher bands while minimizing losses from the first band. At this acceleration and a typical well depth of $V_0/h=102$ kHz, the LZ expression for the lifetime of the first and second bands yields 18 s and 87 μs, respectively [9]. Therefore, only the first band was significantly bound to the potential during this acceleration, and the atoms occupying it were transported to a velocity of 1.2 m/s separating them in velocity from the other atoms.

After separating in velocity the subset of our atoms in the fundamental band from the MOT distribution, the acceleration was changed to a specific value in the range 600 m/s^2 to 1700 m/s^2, and the strong and weak phase modulations were turned on smoothly during 16 μs to avoid phase jumps that could drive transitions between bands. The total time that the atoms were exposed to the acceleration and phase modulation was 500 μs. During this period, the weak probe could, if on resonance, induce interband transitions and deplete the first band.

To determine the depletion of the first band, an acceleration identical to the first was imposed to separate in velocity those atoms still in the first band from those that made transitions to higher bands. After a 3-ms free drift, the resulting spatial distribution was ‘‘frozen’’ in place for 10 ms by an optical molasses; during this phase the fluorescence was imaged onto a charge-coupled-device camera. We observed three spatially resolved groups of atoms that correspond to the MOT distribution left behind during the first acceleration, those atoms that were driven out of the first band during the intermediate acceleration and modulation, and those atoms that survived the interaction, remained in the lowest band, and were accelerated to the final velocity. The fraction remaining in the first band was obtained by normalizing the fluorescence signal from the survivors by the total fluorescence of atoms initially prepared in the first band.

The first set of results presented constitute our study of the normal Wannier-Stark ladder, with only an acceleration and the weak probe modulation present. The probe resonance condition for driving a transition from a WS state in first band to one in the second band is

$$\omega_p=\frac{\overline{E}_g}{\hbar}+m\omega_B, \quad (2)$$

where m is an integer and

$$\overline{E}_g=\frac{1}{2k_L}\int_{-k_L}^{k_L}[E_1(k)-E_0(k)]dk \quad (3)$$

is the energy separation of the first and second bands averaged over the first Brillouin zone [10]. From Eq. (2) we see that the spectrum is characterized by a series of resonances spaced by the Bloch frequency and centered at a frequency corresponding to the average band spacing.

Figure 1 shows three measured spectra for the accelerations of (947, 1260, and 1680) m/s^2 that correspond to the Bloch frequencies $\omega_B/2\pi=$ (16.0, 21.4, and 28.5) kHz, respectively. The spectra were obtained at a fixed well depth of $V_0/h=91.6$ kHz and a fixed probe modulation amplitude of $\delta_p/\omega_p=0.05$. The accuracy of the well depth determination was limited to an uncertainty of $\pm10\%$ by our measurement of the absolute intensity of the interaction beams; however, the shot-to-shot fluctuations were maintained below the level of $\pm2\%$ by monitoring the power using calibrated photodiodes. For a well depth of $V_0/h=91.6$ kHz, the average band spacing is $\overline{E}_g/h=104$ kHz which is in good agreement with the location of the central resonance in the three spectra of Fig. 1. While Eq. (2) predicts the location of the spectral features, it is simply a statement of energy conservation and does not include the attenuation of the transition probability for states lying outside the allowed energy bands.

Also shown in Fig. 1 is the result of a numerical integration of the time-dependent Schrödinger equation using the

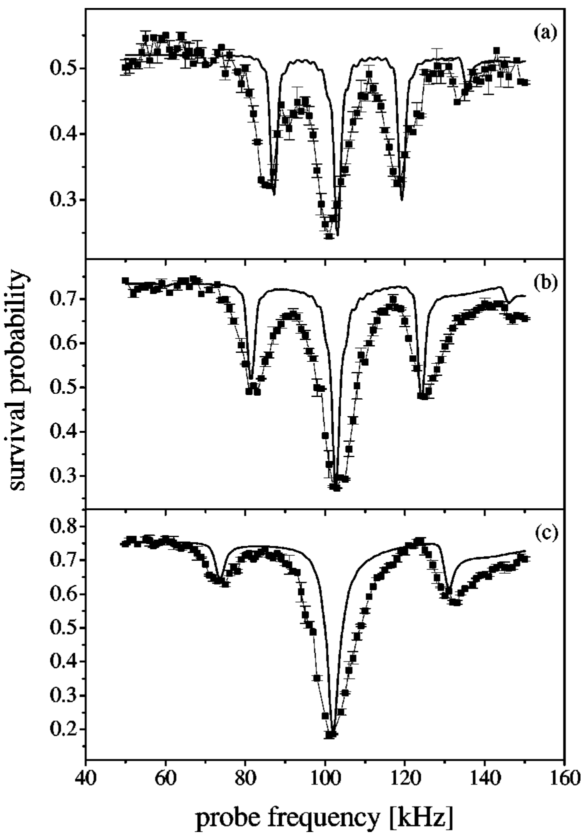

FIG. 1. Wannier-Stark ladder resonances for a well depth of $V_0/h = 91.6$ kHz and accelerations of (a) 947 $\mathrm{m/s^2}$, (b) 1260 $\mathrm{m/s^2}$, and (c) 1680 $\mathrm{m/s^2}$, which correspond to the Bloch frequencies $\omega_B/2\pi = 16.0$, 21.4, and 28.5 kHz, respectively. For the chosen well depth, the average band spacing is $\overline{E_g}/h = 104$ kHz, which is in good agreement with the location of the central resonance. The points are connected by thin solid lines for clarity. The thick solid line shows the results of numerical simulations using the experimental parameters.

experimental parameters. We believe that phase noise in the interaction beams prevented the survival probability from reaching unity when the probe was far from resonance and reduced the depth of the spectral features by a constant factor. For this reason, the y values of the theory curves were shifted and scaled to match the baseline and amplitude of the central resonance. In addition, the value for the probe modulation amplitude was adjusted from 0.05 to 0.035 to reproduce the relative peak heights. This systematic shift was also accounted for in the spectra of the fractional resonances of Fig. 3.

The spectral width of the resonances is fundamentally determined by the finite lifetime of the WS states due to tunneling. For the case of $a = 1680$ $\mathrm{m/s^2}$, where the tunneling rate is the highest, the LZ lifetime of the states in the second band exceeds 125 μs, leading to a broadening of less than 1.3 kHz. Due to the finite probe interaction time of 500 μs, one would expect the resonances to be further broadened by 2 kHz; however, there were a number of experimental mechanisms that contributed to increasing the measured width beyond that predicted by the simulations. The first source of line broadening was due to phase noise in the standing-wave beams with a 5-kHz bandwidth resulting from mirror vibrations and electronic noise in the acousto-optic

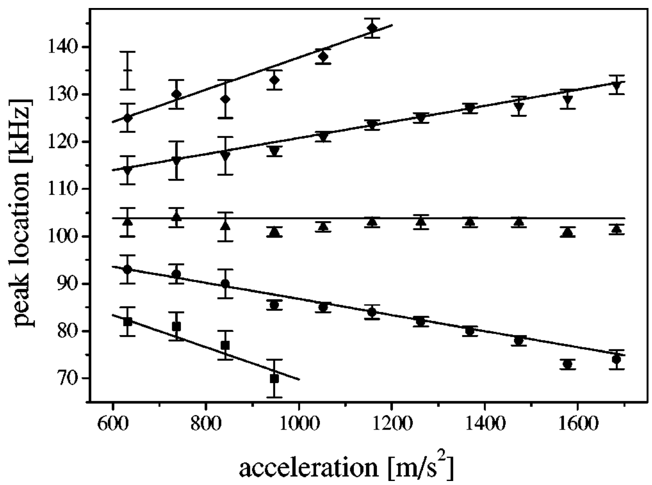

FIG. 2. A fan plot of the Wannier-Stark resonances for a well depth of $V_0/h = 91.6$ kHz. The position of the center resonance is independent of acceleration and corresponds to the average band spacing. The solid lines represent the resonance locations as predicted by Eq. (2).

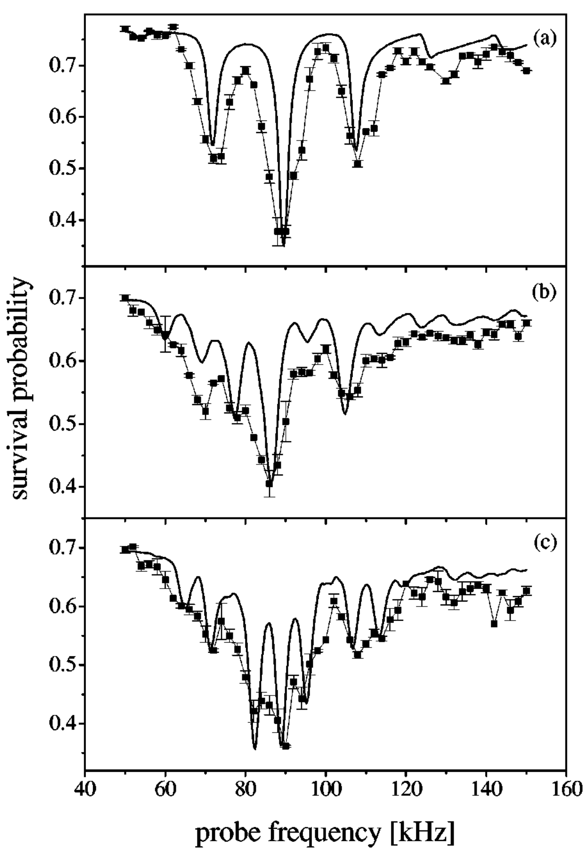

FIG. 3. Fractional Wannier-Stark ladders for a well depth of $V_0/h = 80$ kHz, an acceleration of $a = 1052$ $\mathrm{m/s^2}$, and three values of Ω: (a) $\Omega = 0$, (b) $\Omega = \frac{1}{2}\omega_B$, and (c) $\Omega = \frac{2}{3}\omega_B$ where the Bloch frequency is approximately 18 kHz. The points are connected by thin solid lines for clarity. The thick solid line shows the results of numerical simulations where the well depth was tuned within the experimental uncertainty to match the central peak location. Evidence of peaks developing between the normal Wannier-Stark resonances at a spacing of $(\frac{1}{2}, \frac{1}{3}) \times \omega_B$ can be seen by comparing plots (b) and (c) with (a).

odulator drivers. The second source of broadening came from variations in the well depth, which, for the range considered, is approximately proportional to the average band spacing and therefore the absolute position of the resonances. Although the transverse Gaussian width of the standing-wave beams was large compared to the initial size of the atomic distribution, fast atoms could move radially out of the center and across the profile, producing a time-dependent variation in their effective well depth. By limiting the binning window of the two-dimensional (2D) images in order to restrict our measurement to a colder subset of atoms in one of the transverse directions, we were able to reduce but not totally eliminate this effect.

A series of WS spectra were taken at different accelerations while keeping the well depth constant. The peak locations were determined in each case and are plotted versus the acceleration in Fig. 2, generating the WS fan. The solid lines are the resonance locations as predicted by Eq. (2), and in accordance with theory there is one resonance ($m=0$) at a frequency corresponding to the average band spacing that does not change as the acceleration is varied, while the other peaks spread out with a slope proportional to m.

The second set of results presented constitute our study of the fractional Wannier-Stark ladder, which results when the normal ladder is dressed by a strong ac drive. The ac drive introduces a third time scale into the problem, the first two being the Bloch period and the lifetime of the WS states. The behavior of the WS ladder can be characterized by the electric matching ratio, Ω/ω_B. If this ratio of the strong ac drive frequency to the Bloch frequency is equal to a rational number $\Omega/\omega_B=q/p$, then the resonance condition Eq. (2) becomes

$$\omega_p=\frac{\overline{E_g}}{\hbar}+r\left(\frac{\omega_B}{p}\right), \quad (4)$$

where $r=(mp-lq)$ is an integer and l is the number of participating photons from the strong drive. From conservation of energy, it is clear that the resulting ladder of resonances has a spacing p times smaller.

Figure 3 shows a series of three spectra taken at three different values for the strong modulation frequency $\Omega=(0,8.5,11.3)$ kHz corresponding to $(0,\frac{1}{2},\frac{2}{3})\times\omega_{\mathrm{B}}$. All three spectra were taken at an acceleration of 1052 m/s^2, a well depth of $V_0/h=80$ kHz, a probe modulation amplitude of $\delta_p/\omega_p=0.05$, and a strong modulation amplitude of $\delta/\Omega\approx2.5$. Clearly, the limiting factor is the linewidth broadening described in the preceding paragraph; however, evidence of fractional peaks forming at a spacing of $\frac{1}{2}$ and $\frac{1}{3}$ of the normal resonance spacing can be seen by comparing plots (b) and (c) with (a). The visibility of the fractional peaks was a sensitive function of the strong modulation amplitude and frequency. The results of the numerical simulations are shown by the thick solid lines. To match the central peak location to the experimental data we tuned the well depth within our experimental uncertainty of $\pm10\%$ and the probe modulation amplitude of $\delta_p/\omega_p=0.035$ was chosen as previously described.

In summary, we have observed the Wannier-Stark Fan and fractional Wannier-Stark ladder resonances by direct probe spectroscopy of the band structure of atoms in an accelerating optical potential. Future improvements of the accelerating lattice phase stability should enable the resolution of the natural linewidths of these metastable resonances, and a determination of their lifetimes.

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