Transition to Parametric Wave Patterns in a Vertically Oscillated Granular Layer

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Experiments on vertically oscillated granular layers reveal, at a critical acceleration, a well-defined transition from a flat surface to standing wave patterns oscillating at half the excitation frequency. The patterns observed in a cylindrical container are squares or stripes; a continuous transition between these states occurs when the vibration frequency is varied at constant acceleration. The dispersion relation for these parametric granular waves is similar to that for gravity waves in a fluid, but exhibits a well-defined cutoff associated with the particle size.

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Propagating wave fronts, solitons, and extended spatial patterns are examples of the collective behavior often observed in systems driven far from equilibrium [1]. In continuous systems the spatial coupling, which is a macroscopic manifestation of molecular interactions, is responsible for the collective motion. Collective behavior has been observed recently in granular materials [2–6], but in contrast to continuous systems, the spatial coupling arising from the interactions between the grains is not well understood. Difficulties arise because granular materials comprise a unique state of matter with properties common to both fluids and solids [7]. These shared properties make the application of statistical methods to the definition of mean quantities difficult, and cause phenomenological coefficients, such as viscosity or elasticity, to be strongly singular [7]. Despite these formidable challenges, the significant role of granular dynamics in industry and geology makes the understanding of these materials an important subject for science and technology.

Our experiment examines collective phenomena in thin layers of glass particles oscillated vertically. Despite the similarity of the observed patterns to parametrically excited surface wave patterns in fluid layers [8], the granular system is quite different: The waves we observe do not represent a parametric excitation of known intrinsic collective modes of the granular layer; the waves exist only in the presence of sustained external forcing [9]. This behavior stands in sharp contrast to that of fluid systems where the intrinsic modes (gravity waves) persist for long times. However, we find that the dispersion relation for waves in granular layers is similar to that for gravity waves in a fluid, including a well-defined minimum wavelength like that associated with dissipation in a fluid [10]. These observations suggest that the collective mode we observe is a normal mode strongly damped by internal granular friction. The spatial coupling appears as a result of a lateral momentum transfer associated with multiple collisions among the grains.

Our experiments are conducted in a cylindrical Plexiglas cell (76 mm in diameter, 40 mm in height) filled with glass particles to a depth of 3–100 particle diameters. The particles have diameters of 0.2, 0.3, or 0.4 mm. The cell is mounted with its principal axis parallel to gravity on a Ling-Electronic vibration exciter driven by a frequency synthesizer (10–100 Hz). The amplitude of the sinusoidal acceleration, one of the bifurcation parameters, is measured with a Wilcoxon Research 111 accelerometer and a B&K 2635 charge amplifier. A stroboscope, synchronized at half the driving frequency, removes the horizontal motion associated with the parametric waves and allows the dynamics of the pattern to be followed. We use as parameters the dimensionless layer thickness, $N=H/D$ (where H is the layer thickness and D the particle diameter), and the dimensionless acceleration amplitude $\Gamma=4\pi^2f^2A/g$ (where f is the driving frequency, A is the driving amplitude of the cell, and g is the acceleration of gravity).

Six photographs of patterns observed above the instability threshold at Γ_c are shown in Fig. 1. For small f and increasing Γ, the flat surface bifurcates to a square pattern consisting of two standing waves with perpendicular wave vectors oscillating at half the driving frequency [Fig. 1(a)]. For f larger than a critical value f_c, the pattern takes the form of stripes consisting of a single parametric standing wave [Fig. 1(b)]. The stripes appear to be constrained to be perpendicular to the wall of the container, which can lead to strong curvature near the wall. These patterns have many of the features observed in other systems such as Raleigh-Bénard convection in a cylindrical container [11]. For example, new stripes nucleate in regions of high curvature and defects are generated in the center of the cell where the wavelength is smaller [Figs. 1(c) and 1(d)].

The instability threshold, determined by increasing Γ with H and f fixed, is shown as a function of Γ and f in Fig. 2. The transitions are subcritical and the associated hysteresis is typically 5%; the hysteresis increases with decreasing f or N.

No model exists that can adequately describe a granular system [7]. However, a possible explanation for the instability mechanism is provided by the following simple argument. For acceleration amplitude larger than g, at the point of the cycle where the effective gravity becomes negative, the layer loses contact with the base of the con-

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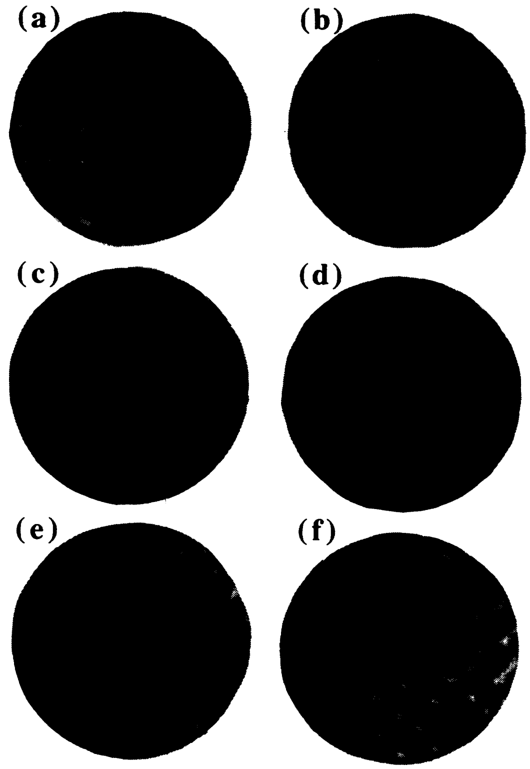

FIG. 1. Patterns observed close to the instability threshold: (a) square pattern ($f=23.2$ Hz, $\Gamma=3.5$); (b)-(d) striped patterns with some dislocations ($f=27.6$ Hz, $\Gamma=4.23$); (e),(f) disordered patterns near the squares-to-stripes transition ($f=25.1$ Hz, $\Gamma=4.3$) (depth of layer, 1.75 mm; particle diameter, 0.2 mm).

tainer, creating a small gap at the bottom of the layer. The momentum transfer due to the subsequent collision of the layer with the cell bottom is observed both in the signal of a transducer in contact with the base of the container and in the cell accelerometer signal. These signals are used to determine the collision duration δt and the flight time Δt [5]. When Γ is slightly larger than 1, $\Delta t \ll 1/f$ (the excitation period), but when Γ is increased to a critical value $\Gamma=\Gamma_c$, Δt becomes equal to $1/f$. Then, for $\delta t \ll \Delta t$, which occurs when f is small, the collision of the layer takes place when the acceleration of the plate is close to $-g$, i.e., close to the takeoff point. Because the collision is not instantaneous, the particles colliding first acquire the velocity of the plate and then lose contact with it before the collision is completely finished. The interaction of the subsequent rising and falling momentum fluxes provides a possible mechanism for the instability.

The picture described above is supported by experiments performed at low frequency and small layer thickness [12]. An estimate of Γ_c can be obtained by considering the one-dimensional motion of a single particle colliding inelastically with an oscillating surface. A straightforward calculation gives $\Gamma=\Gamma_c^{sp}=(\pi^2+1)^{1/2}=3.30$, in good accord with the average observed value at small f of 3.35 (see Fig. 2). Moreover, the collision observed in the accelerometer signal occurs near the point where gravity vanishes.

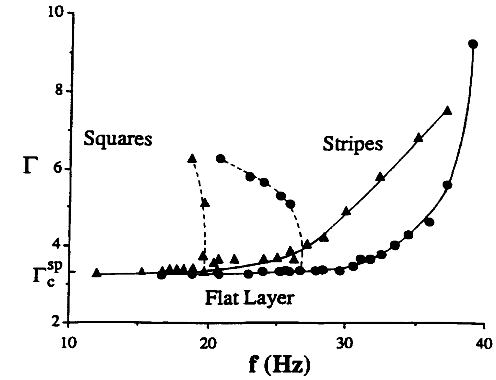

FIG. 2. Stability diagram showing the transition from a flat layer to squares or stripes (solid lines) as a function of the dimensionless acceleration Γ and driving frequency f, for layer depths of 1.75 mm (●) and 4.3 mm (▲) (for particles of diameter 0.2 mm). The dashed lines separate regions of squares and stripes, and Γ_c^{sp} is the instability onset for a single particle (see text).

If the layer thickness or the frequency is made large, the collision time of the layer δt becomes large [12]. Then not all the particles collide with the bottom or lose contact with it simultaneously. We speculate that the particles closest to the free surface of the layer take off first and collide last [13]. At $\Gamma=\Gamma_c^{sp}$, only the few particles nearest to the free surface of the layer can give a negative momentum contribution at the point of effective zero gravity. Thus, Γ has to be increased above Γ_c^{sp} in order that more particles can contribute to the negative momentum flux. The strong increase in the instability threshold is thus plausibly explained by the spreading in the collision time of the layer. In addition, because the spreading is negligible when $\Gamma g/4\pi^2 f^2 \delta H > 1$ (where δH is the maximum dilation of the layer) [12], this strong increase should appear at smaller frequencies for thicker layers, as is observed experimentally (see Fig. 2).

The mean wavelength λ of the pattern close to the onset of squares is plotted in Fig. 3(a) as a function of H for three different particles sizes. The wavelength first increases with H but then saturates at a value that is almost independent of D. The location of the saturation point increases with D, but corresponds to a constant value of $N(\approx 7)$. Note that the waves disappear when the layer is only a few particle diameters thick ($N \approx 3$). Figure 3(b) shows the dependence of λ on f near the onset of instability for $N=7$ and three different particle sizes. For the three values of D studied, the data indicate that $\lambda \propto 1/f^2$ and that λ exhibits a well-defined minimum λ_{min}, when f goes to infinity [see Fig. 3(b)]. The slopes of the straight lines in Fig. 3(b), g_{eff}, are insensitive to changes in N for $N > 7$. However, in the region where λ is a function of H, the slopes are a slowly increasing func-

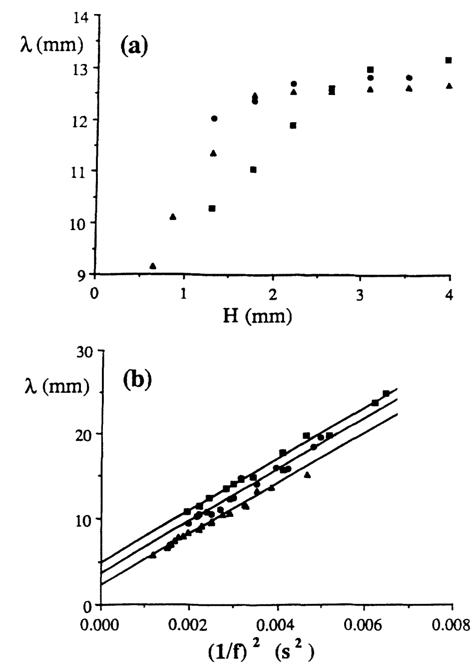

FIG. 3. Wavelength dependence on (a) layer depth (with $f=18.3$ Hz) and (b) excitation frequency (with $N=7$), for particles with diameters 0.2 mm (▲), 0.3 mm (●), and 0.4 mm (■) (with $\Gamma=3.5$). The intercepts yield the minimum wavelength, λ_{min}.

tion of N [see Fig. 3(a)]. The largest changes in g_{eff} were obtained for particles with diameters equal to 0.4 mm and were on the order of 20% when N was increased from 3 to 7. These changes in N caused no appreciable changes in λ_{min}, indicating that it is only a function of D, $\lambda_{min}=\lambda_{min}(D)$. For $N\geq 7$, the observations are summarized by the dispersion relation

$$\lambda-\lambda_{min}(D)=g_{eff}/f^2\,, \quad (1)$$

where $\lambda_{min}(D)\approx 11D$ and $g_{eff}\approx 310$ cm/s^2.

The observation that $g_{eff}\sim g$ is consistent with a gravitational restoring force giving rise to parametric waves in the granular layer. Moreover, Eq. (1) is similar to the dispersion relation for surface gravity waves [14]. The wavelength must be related to both the lateral velocity of the grains, V_l, and the time available for the lateral motion of grains, t_l. Note that a lateral transfer of mass is necessary within each period of the excitation in order to sustain a stationary wave. Moreover, associated with each mass transfer is a lateral momentum transfer that is due to the lateral gradient of the vertical momentum of the particles [15]. This effect is analogous to the pressure gradient giving rise to lateral motion of fluid in gravity waves. Because the lateral motion of grains occurs when the layer collides with the bottom of the container, V_l must be a fraction of the collision velocity, V_c, and t_l must be on the order of the ratio a/V_c, where a is the amplitude of the pattern. Thus, λ must be proportional to a. This proportionality is supported by the experiments, with a proportionality constant equal to 0.6. In addition, measurements of the mean wavelength of the square pattern, performed with D, H, and f fixed, show that λ increases linearly with a. Thus, at constant Γ, λ must be proportional $1/f^2$ and the proportionality constant must be on the order of g, as is observed experimentally.

Recent experiments on the parametric instability of a liquid-vapor interface close to the instability onset found that the wavelength of the patterns exhibited a cutoff associated with dissipation [10]. By analogy, the present observations of a minimum wavelength suggest that the waves we observe are strongly damped by internal friction; at λ_{min} they are too strongly damped to be parametrically amplified. Thus, since λ_{min} depends on D [cf. Fig. 3(b)], our interpretation suggests that the dissipation depends on particle size.

At small frequencies, λ is about one-fourth of the diameter of our cell, so that one might be concerned about errors introduced by the quantization of the wavelength. However, a few experiments conducted in a square container have shown that the shape and size of the cell do not affect the wavelength selection of the pattern.

A continuous transition between squares and stripes is observed by slowly varying f while keeping Γ constant. When f is decreased, a lateral instability of the stripes grows in amplitude, giving rise to the square pattern [see Figs. 1(e) and 1(f)]. This transition is indicated by the dashed lines in Fig. 2. The intersection of the dashed and solid lines defines a characteristic critical frequency for this transition, which is a linear function of H. We remark that the squares-to-stripes transition is not concomitant with the strong increase in the instability threshold observed in the striped pattern region (see Fig. 2). The pattern exhibits intermittent defect nucleation even close to the threshold at Γ_c. When the excitation is increased further, the defect nucleation becomes more frequent and the wave pattern displays a transition to spatiotemporal chaos, as is often observed in pattern forming systems.

In conclusion, our study of parametrically excited granular waves reveals that the wavelength of the pattern is, for thick layers, independent of the layer depth, but varies linearly with $1/f^2$, just as for gravity waves in a fluid. However, the granular waves exist only in the presence of strong sustained external forcing, which suggests that they are heavily damped by internal friction. Measurements of the instability threshold for the flat layer suggest that the wave formation is related to the interaction between two particle fluxes with opposite momenta; an upward flux forms when the falling dilated layer collides with the base of the oscillating container.

Future experiments conducted with particles of different coefficients of restitution and under different pressure conditions should help clarify the roles of particle elasticity and interstitial gas viscosity in the spatial coupling of the layer.

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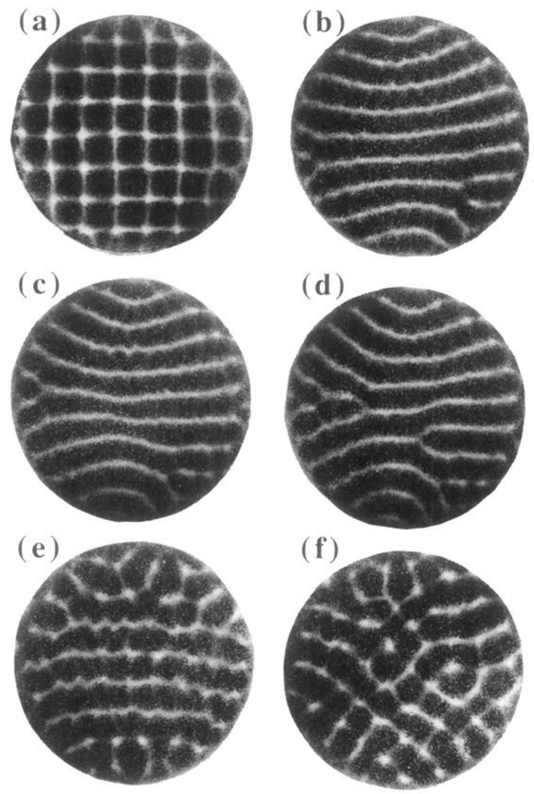
[13] Particles in contact with the bottom of the container cannot lose contact first because, in that case, an air flow would be needed to fill the gap left by the relative motion of the layer. The motion of particles close to the free surface does not require such a flow.

[14] See, for instance, L. D. Landau and E. M. Lifshitz, *Fluid Mechanics* (Pergamon, New York, 1984), p. 36.

[15] This interpretation suggests that parametrically excited granular waves should disappear in materials with a very low coefficient of restitution.



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