Comment on "Universal Scaling of the Stress Field at the Vicinity of a Wedge Crack in Two Dimensions and Oscillatory Self-Similar Corrections to Scaling"

In a recent Letter, Ball and Blumenfeld [1] argue that certain self-similar fracture patterns can be understood by examining the stress fields surrounding a wedge. The point of this Comment is that the linear elastic stress fields around a wedge do not display the features that Ball and Blumenfeld emphasize.

In the course of their argument, Ball and Blumenfeld examine the stress fields around an infinite wedge with opening angle $2(\pi-\alpha)$. This problem is known [2] to have solutions which are proportional to r^m. Although for the leading singularity near the wedge tip, m is real, there exist solutions for which m is complex. The authors of the Letter argue that these subleading oscillatory solutions contribute to the creation of self-similar fracture structures for crack assemblies of finite extent. However, there are several cases in which a problem with a finite wedge can be solved, and once one includes boundary conditions at infinity the stress field does not have this type of oscillatory character.

One such problem is that of the stress fields outside of the shape formed by the intersection of two circular arcs. This problem has an analytical solution found by Ling [3], and graphs of the solution do not oscillate. More general shapes can be examined by numerical methods. For example, Fig. 1 presents pictures of the stress outside a wedge described by the teardrop map

$$z(w)=w(1-1/w)^{1+\beta}, \qquad (1)$$

where w lies in the unit circle in the complex plane. Far from the wedge, one imposes a uniform stress $\sigma_{yy}=\sigma_\infty$. An accurate numerical solution may be obtained by replacing $z(w)$ with $z_N(w)$, its Laurent series up to order w^{-N}. Then one has a type of problem that can be solved by the methods of Muskhelishvili [4]. Ball and Blumenfeld [5] have pointed out that the most interesting thing to examine is the stress $\sigma_{yy}+\sigma_{xx}$ approaching the tip along the top surface of the teardrop. This principal stress is shown as the lower inset of Fig. 1 for $N=999$, $\sigma_\infty=4$, $\beta=0.5$ (so that Ball and Blumenfeld's $2\alpha/\pi$ equals 1.5). Small oscillations can be detected, and are brought out in the main portion of the figure, but they are periodic in r, rather than $\ln r$, and have a wavelength proportional to the radius of curvature of the wedge tip [6].

There is no numerical evidence that oscillating eigenfunctions for the infinite wedge problem are important for coarse-grained assemblies of cracks of finite size. However, Ball and Blumenfeld's surprising assertion that stress fields can oscillate along the faces of a wedge appears to be correct if effects related to the finite curvature of the wedge tip are taken into account.

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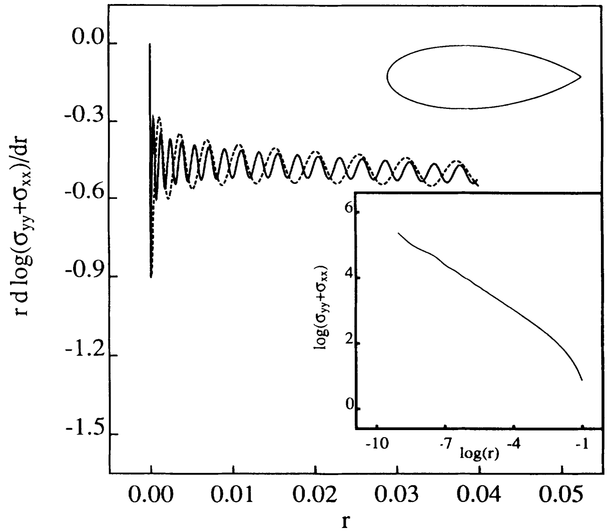

FIG. 1. The lower inset plots the logarithm of principal stress vs the logarithm of arclength r from the wedge tip along the top surface of the shape depicted in the upper right-hand corner of the figure. The shape is produced by the first $N=1000$ terms in the Laurent series of Eq. (1), with $\beta=0.5$, and a stress at infinity $\sigma_\infty=4$. The leading power-law singularity is in accord with predictions for an infinite wedge. The small oscillations visible in the inset are emphasized by differentiating the principal stress in the main part of the figure. The solid line corresponds to $N=1000$, and the dashed line to $N=500$; note that the oscillations are essentially periodic in r, not $\ln r$, and that their length decreases with the wedge tip radius.

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[4] N. I. Muskhelishvili, *Some Basic Problems in the Mathematical Theory of Elasticity* (Noordhoff, Gronigen, 1952), Sect. 84. Numerical routines from W. Press *et al.*, *Numerical Recipes* (Cambridge Univ. Press, New York, 1988), were employed.

[5] An earlier version of this Comment examined stresses along the line $\theta=0$; in their preliminary response, Ball and Blumenfeld correctly pointed out that logarithmic oscillations should be damped strongly in that direction.

[6] Numerically one finds that the radius of curvature of the tip scales as $N^{-1.5}$. For $N=1000$, the radius is $\sim 3\times10^{-5}$—much smaller than the oscillations of Fig. 1.

