Instability in Dynamic Fracture

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Cracks in brittle materials have terminal velocities far below theoretical predictions. To address this problem we have investigated the propagation of cracks in a brittle plastic (polymethylmethacrylate). Velocity measurements with resolution an order of magnitude better than past experiments reveal the existence of a critical velocity at which the velocity begins to oscillate, the mean acceleration drops sharply, and a pattern is formed on the fracture surface. Thus the dynamics of cracks may be governed by a dynamical instability.

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Many recent problems in physics concern the connection between structure and dissipation. Complicated patterns arise as systems shed their excess energy, and a detailed understanding of those patterns is necessary if one is to explain precisely how the dissipation occurs. A problem unfamiliar to most physicists [1] where such a line of reasoning should prove profitable is the study of fracture dynamics in brittle materials [2,3].

Although much work has been done in the field in over seventy years, the mechanisms that govern the dynamics of cracks are not well understood. The most obvious difficulty is that cracks do not attain the limiting velocity predicted by linear elastic theory [4]. In this Letter we will present evidence that crack dynamics in brittle materials are governed by dynamical instabilities of the crack tip. This observation of an instability in our data provides a new context in which to view many previously uncorrelated experimental observations.

This premise enables us to draw together a number of previously uncorrelated experimental observations into a new picture of instability-driven fracture dynamics.

Analysis of energy flow into the crack tip for the geometry of Fig. 1(a) predicts that a crack should smoothly accelerate until reaching the Rayleigh wave speed [2], the speed at which sound travels across a flat surface. In reality, cracks travel more slowly. For example, in the brittle plastic polymethylmethacrylate (PMMA), the subject of our experimental investigations, the Rayleigh wave speed is 975 m/s, but cracks have rarely been observed to surpass six-tenths of this speed [4(c)]. Theories of crack motion are based on the assumption that cracks may be treated as smooth, straight, and traveling at nearly constant velocity, but fracture surfaces are ragged, complicated objects with structure over a wide range of length scales [4-6].

Fracture surfaces in brittle materials exhibit a characteristic pattern sequence known as [6] "mirror, mist, hackle": An initially smooth and mirrorlike fracture surface begins to appear misty, and then evolves into a rough hackled region. In some brittle plastics, such as PMMA of high molecular weight, the pattern can also exhibit a characteristic wavelength [7,8]. Surface roughness is known to increase with crack speed [9,10], and periodic stress waves have been observed in high-speed photographs to be emitted from the tips of rapidly moving cracks in a wide variety of materials [11]. Finally, at a velocity of ~400 m/s in PMMA a large increase is observed in energy flux to the crack tip [9].

Phenomenological "dissipation" has been invoked in order to explain the discrepancies between theory and experiment regarding crack velocity, but no mechanism for this dissipation has been proposed. By careful measurement of crack velocity and fracture surface profile we will show that previously undetected violent oscillations in the velocity begin once the mean velocity passes a well-defined critical value. These oscillations are well correlated with spatial structure on the newly created crack surface, and immediately following their onset the system abruptly changes its mean acceleration. These results indicate that the behavior of cracks may be understood as a dynamical system exhibiting bifurcations that should be amenable to theoretical analysis [12].

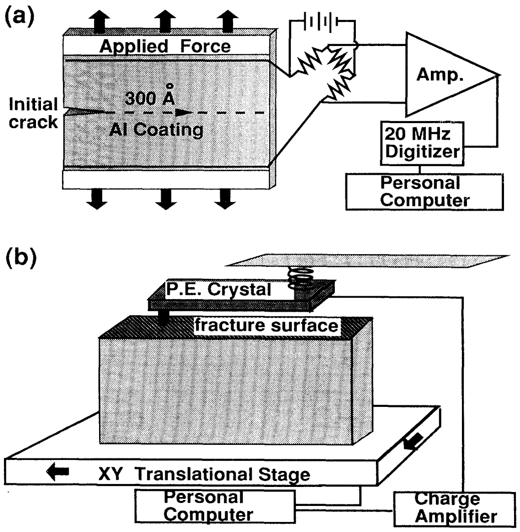

FIG. 1. Schematic representation of the experimental apparatus: (a) velocity measurements; the path of the crack (dotted line) and its direction of propagation are indicated. (b) Surface profile measurements.

Experimental methods.— Our experiments are performed on thin (1.6 or 3.2 mm thick) sheets of Plexiglas (PMMA) [13]. The samples have vertical dimensions of 20–25 cm and range from 10 to 20 cm in the horizontal direction (defined as the direction of crack propagation). The quasi-two-dimensional geometry is chosen so that our results can be compared with two-dimensional elastic theory. The experimental setup is shown in Fig. 1(a). We seed a small horizontal crack at the edge of the sample, typically 3–4 mm in length. We apply uniform stress to rigid tabs fixed at the top and bottom of the sample using a computer-controlled tensile-testing device that enables us to apply any rate of strain from 0 to 150 mm/min in displacement increments of 0.1 μm.

Starting at approximately half of the critical strain needed for the onset of fast fracture [14], we increase the strain every 10–20 s by a fixed displacement δl (typically $\delta l/L \sim 1\times10^{-4}$, where L is the vertical length of the sample) until the sample breaks. The rate of strain is chosen so as to have at most one 0.1-μm step during the course of the fracture; thus we eliminate elastic waves induced by sample loading. The applied stress is continuously monitored at 25 kHz by means of a load cell; stresses of 0–25000 N can be measured with a resolution of 0.05%.

We measure the crack tip location as a function of time by means of a thin (250 Å) aluminum layer evaporated onto one face of the sample. The layer is thin enough to have a negligible effect on the mechanical properties of the sample. The plate resistance, which increases continuously with the crack length, is used as one leg of a Wheatstone bridge whose output is amplified and then digitized to 8 bits at 20 MHz. In order to capture the entire length of the crack at high spatial resolution, four digitizing channels are cascaded with different gains and offsets, each channel being triggered as the previous one exhausts its range. The plate resistance as a function of crack length is calibrated numerically for each sample geometry used. This calibration has been checked carefully by direct measurements of the resistance of plates with stationary cracks. With this method we are able to measure velocity with a resolution of 25 m/s and a mesh of 0.2 mm across the entire length of the sample— an order-of-magnitude improvement over previous results obtained by nonintrusive methods [15].

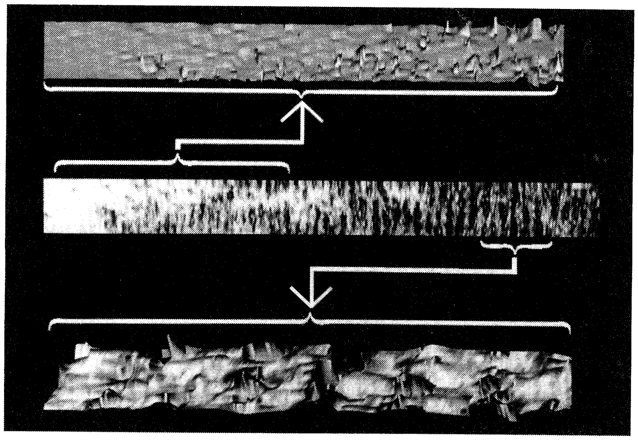

FIG. 2. Computer visualizations of the profilometer data for a crack that propagated from left to right. The central image shows an overview of a portion of the fracture surface, 62 mm × 1.5 mm. Lighting models are used, so that the image is nearly identical to illuminated photographs of the surface. Note the ripple pattern with wavelength on the order of 1 mm. Two subregions have been magnified and are shown in perspective. The onset of the instability appears in the upper image; the highest peaks are about 20 μm. The lower image contains a magnified view of the ripples created by the instability once it develops more fully; the highest peaks are about 50 μm.

In order to measure quantitatively the amplitude of the new surface created by the crack as a function of position, an x-y scanning profilometer has been constructed, as shown in Fig. 1(b). The sensor is a piezoelectric crystal sandwiched between a stylus and a leaf spring. The stylus tip is deflected as the sample is translated on a computer-controlled translational stage. The resolution in the x-y direction is 25 μm and is limited by the radius of the stylus. Deflections of the stylus of 0–1 mm normal to the crack surface can be measured with 0.1-μm resolution.

Results.— A fracture surface profile determined from the profilometer measurements is shown in Fig. 2. The surface is essentially featureless up to a certain point. Suddenly a jagged structure appears and develops into oscillations with a wavelength (about 1 mm) that is unchanged when plate thickness is increased from 1.6 to 3.2 mm. Since PMMA is completely amorphous on this scale [16], the source of these oscillations is not related in any obvious way to the material structure [17].

The appearance of surface structure correlates with a marked change in the behavior of the crack velocity, as

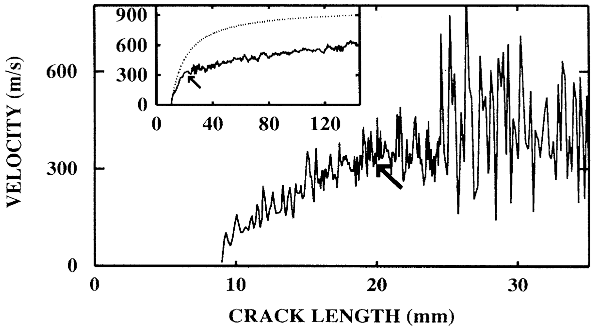

FIG. 3. At the critical velocity of 330 ± 20 m/s, indicated by an arrow, the mean acceleration of a crack slows, and the velocity begins to oscillate. These oscillations are much larger than the resolution. Inset: The measured velocity of the crack, averaged on the scale of 1 mm (solid line), compared with the theory of Freund (dotted line), Ref. [2]. The theory should apply strictly up to 30 mm, and predicts an asymptotic velocity of 975 m/s.

Fig. 3 illustrates. Initially the crack accelerates smoothly and rapidly in the region where the fracture surface is smooth (the mirror regime). Then at a well-defined critical velocity, which is independent of sample geometry, sample thickness, applied stress, and acceleration rate of the crack tip, the mean acceleration slows sharply and the velocity begins to oscillate. The appearance of the oscillations coincides with the appearance of jagged structure on the fracture surface, as shown in Fig. 2.

The strong correlation between velocity and surface oscillations is illustrated by Fig. 4(a), which shows a typical cross-correlation function between the oscillatory part of the velocity and the surface height. The velocity and surface oscillations have a well-defined period as a function of time—the spatial wavelength increases with the mean velocity. Evidence that the oscillations have a well-defined temporal rather than spatial frequency is presented in Fig. 4(b), which shows temporal power spectra of surface deviations (averaged across the thickness of the sample) for two consecutive regions of a crack surface. In these regions the average velocity changes by over 60%, but the value of the oscillation frequency is constant (about 360 kHz).

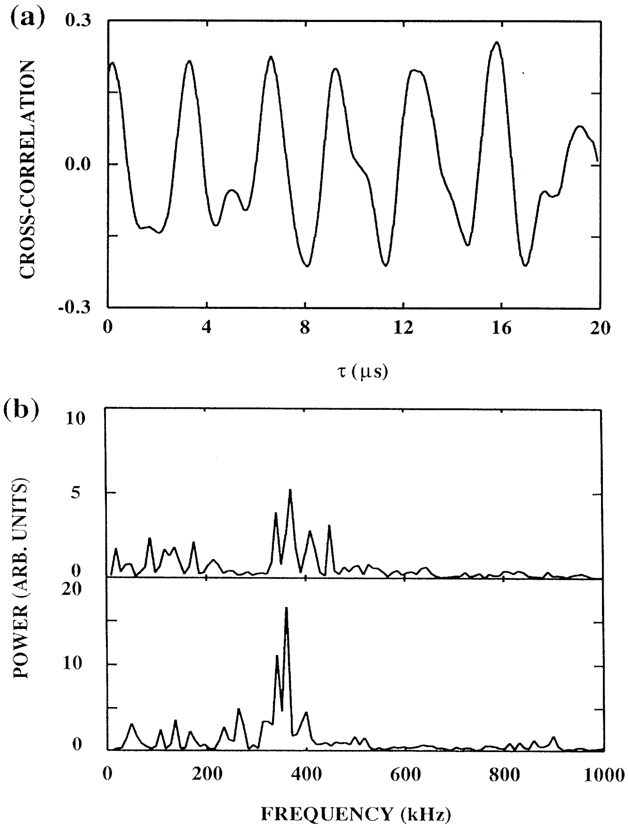

FIG. 4. (a) The correlation of the velocity oscillations with surface profile in a region well beyond transition is shown in this graph of the cross-correlation function, $\sum_t A(t)V(t+\tau)$, where τ is the delay time and A and V are, respectively, the surface height and fluctuating part of the velocity, normalized by their rms values. (b) Power spectra of surface height. Upper: Surface created in the 100 μs immediately following the onset of oscillations. Lower: Surface created in the subsequent 100 μs. Although the mean velocity of the crack increases by over 60%, the frequency of oscillation remains constant.

Evidence of an invariant critical velocity for the onset of the oscillations is given in Fig. 5, which shows crack velocity as a function of the crack length at the onset of the oscillations. The critical velocity has a value of 330 ± 20 m/s. The data are plotted as a function of crack length at the initiation of the oscillations in order to emphasize that the critical velocity is not influenced by effects such as the interaction of the crack with sound waves induced by the fracture initiation and subsequently reflected by the lateral or vertical boundaries. These data were obtained in both extruded and cell-cast PMMA samples of thickness 1.6 and 3.2 mm, ranging in size from 10×25 to 20×25 cm. Runs were conducted in air, helium, and nitrogen atmospheres [18]. The acceleration rates of the cracks in the various runs range from 1×10^6 to 1×10^7 m/s^2, and the applied stress per unit area required for fracture ranged from 4.8 to 17 MPa.

We have presented experimental evidence indicating the existence of a dynamical instability in brittle fracture of PMMA. The instability initiates at a critical velocity, controls the subsequent mean crack speed, creates surface structure, and causes dissipation [9,19]. A theory for the instability does not yet exist, but we offer several observations. Figures 2 and 4(a) offer some hope that the basic instability can be understood within two-dimensional elastic theory, although the fracture surface is not simply one dimensional. There is a precedent for such an instability in the calculation by Yoffe [20], which shows that a crack moving along a straight line will branch off at an angle beyond a critical speed. In PMMA Yoffe's critical speed is approximately 620 m/s, well above the velocity at which oscillations are observed, so her instability does not

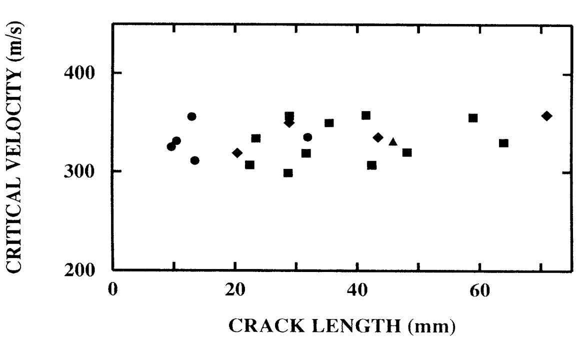

FIG. 5. Critical velocity as a function of crack length at the time of transition. Circles, 1.6-mm extruded PMMA in air; squares, 3.2-mm cell-cast PMMA in air; diamonds, 3.2-mm cell-cast PMMA in helium; and triangle, 3.2-mm cell-cast PMMA in nitrogen. All measurements were at room temperature.

explain our data. The challenge to explain our results theoretically remains open. Many amorphous brittle materials, such as the oxide glasses, exhibit "mirror, mist, and hackle." It remains to be seen experimentally whether the dynamical instability we have observed is generic to brittle fracture.

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[13] The cell-cast plastic is commercially available Rohm and Haas cell-cast PMMA; the extruded plastic is produced by KSH.

[14] There are two dynamical regimes in brittle fracture. The first is a quasistatic regime in which the crack moves less than 1 mm/s. The second is the regime of rapid motion, on the order of hundreds of meters per second, with which we shall be concerned.

[15] Ultrasonic modulation has comparable accuracy, but is intrusive, masking precisely the effects we have observed. See F. Kerkhof, in *Dynamic Crack Propagation,* edited by G. Sih (Leyden, Crows Nest, Australia, 1973), p. 3. Optical and electrical techniques are discussed by H. Bergkvist, Eng. Fract. Mech. **6**, 621 (1974), and A. Kobayashi, N. Ohtani, and T. Sato, J. Appl. Polym. Sci. **18**, 1625 (1974), respectively. B. Stalder, Ph. Béguelin, and H. H. Kausch, Int. J. Fract. **22**, R47 (1983); **23**, R7 (1983), describe a technique similar to ours, using graphite instead of aluminum.

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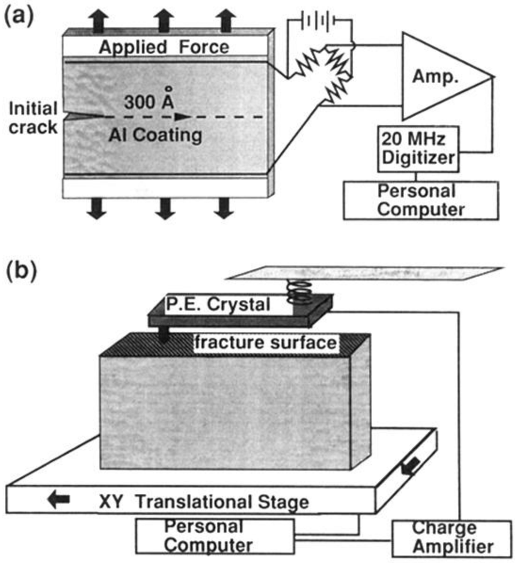

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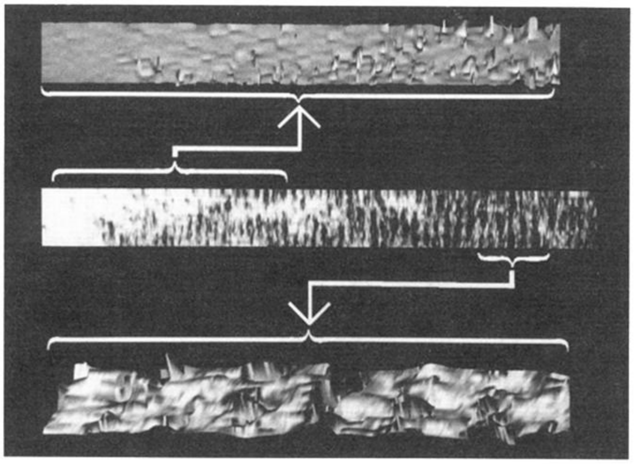

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