Comment on "Initial Stages of Pattern Formation in Rayleigh-Bénard Convection"

In two separate sets of experiments, Meyer, Ahlers, and Cannell[1] gave convincing visual evidence that stochastic effects influence the initial pattern in their Rayleigh-Bénard convection cell. The first set of experiments involved ramping the heat current linearly through the convective threshold, and the second set involved a steady sinusoidal modulation resulting in a reduced Rayleigh number $\epsilon=(R-R_c)/R_c$ of the form $\epsilon(t)=\epsilon_0+\delta\sin\Omega t$. In each case the authors also showed that a simple model with only one adjustable parameter representing the strength of the noise could account for the experimental results. For the ramps, Meyer, Ahlers, and Cannell used the Landau equation

$$\tau_0\partial_t A=\epsilon A-gA^3+kA^5+f, \qquad (1)$$

where τ_0, g, and k are known from other experiments, and f is a Gaussian noise source with fluctuation $\langle f(t)f(t')\rangle=2\tau_0 F\delta(t-t')$. The fit to experiment gave $F=6.03\times10^{-7}$. For the modulation experiments, Eq. (1) could again be used[2] (without the f term) with the condition that if the minimum of $A^2(t)$ over a cycle fell below a critical value A_c^2, the system was stochastic, and if not the system was deterministic. The parameter A_c, representing the noise threshold, was adjusted to give the best fit to the boundary between deterministic and stochastic regions in the ϵ_0,δ plane (see Fig. 1).

In this Comment we wish to point out that the modulation results in Fig. 1 can be accounted for by use of Eq. (1) with *no adjustable parameters, taking the* noise strength F from the fit to the ramp experiments. For $F=\delta=0$ and $\epsilon_0>0$ the solutions of Eq. (1) are on two symmetric attractors. For $\delta\neq0$ and $F>0$ we have simulated Eq. (1) with $\epsilon(t)=\epsilon_0+\delta\cos\Omega t$ and have evaluated the average time t_s between switches from the positive to the negative attractor. If the system switches randomly at each period, we have $t_s^{\min}=\sum_1^\infty n(\frac{1}{2})^{n-1}/\sum_i^\infty(\frac{1}{2})^{n-1}=2$, with t_s measured in periods of the modulation. The calculated t_s obtained after a time t was an irregular function of t for early times, and only settled down to its asymptotic value $\tilde{t}_s(\epsilon_0,\delta)$ after a few thousand periods. For fixed $\delta=0.2$ the dependence of $\tilde{t}_s$ on ϵ_0, shown in the inset of Fig. 1, has a transition from small to large values which we interpret as a transition from random to deterministic behavior. Picking the transition point at $\tilde{t}_s=2t_s^{\min}=4$ we find the value $\epsilon_0(\delta)=0.058$, and similarly other δ values yield the solid curve in Fig. 1. (The bars indicate the spread of values resulting from the choice of transition at $\tilde{t}_s=5t_s^{\min}=10$.) It is clear from the agreement between experiment and theory that the transition from deterministic to stochastic behavior under modulation can be accounted for by the same model as was used for the ramp experiments, without further adjustable parameters, thus providing additional evidence for the consistency of the interpretations given in Ref. 1.

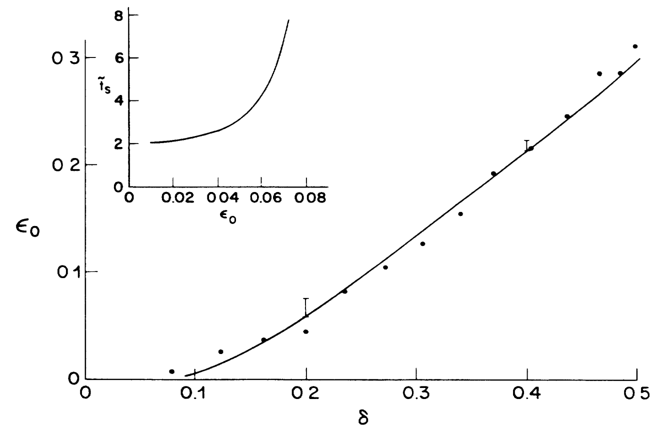

FIG. 1. The boundary between deterministic and stochastic pattern formation under a modulation of the reduced Rayleigh number of the form $\epsilon=\epsilon_0+\delta\cos\Omega t$. The points are the data of Ref. 1 and the line is the prediction of Eq. (1). The bars at $\delta=0.2$ and 0.4 are explained in the text. Inset: switching time $\tilde{t}_s$ vs ϵ_0 for $\delta=0.2$.

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[1]C. W. Meyer, G. Ahlers, and D. S. Cannell, Phys. Rev. Lett. **59**, 1577 (1987).

[2]The theoretical curve in Fig. 5 of Ref. 1 came from a Lorenz model, but according to the authors (private communication) a similar fit is obtained from Eq. (1).

