Argoul *et al.* Reply: The work commented on[1] contains the first numerical evidence that small-mass diffusion-limited aggregates (DLA) are self-similar fractals with generalized fractal dimensions $D_q = 1.60 \pm 0.02$, independent of q. This analysis was performed using box-counting ($q \geq 0$) and fixed-mass ($q < 0$) algorithms. The Comment[2] suggests that the difference between this value of D_q and the well established value of the dynamical dimension $D_d = 1.71 \pm 0.02$, obtained from the dependence of the radius of the gyration on the cluster size, results from finite-size effects.

We have extended our analysis to cluster sizes ($M \sim 5 \times 10^4$) comparable to the range of mass of the off-lattice clusters investigated in Ref. 2. Surprisingly, despite anisotropic effects induced by the underlying square lattice, our results still agree with our previous estimate.[1] In Fig. 1(a) we have plotted the partition function $Z_{q=0}$ vs ϵ on log-log scales. The local slope of this graph gives the local dimension shown in Fig. 1(b); this dimension remains clearly below D_d over the entire accessible ϵ range, although it displays an increasing behavior from small to large scales. This drift probably arises from finite-size effects. In Fig. 1(c), we have plotted the residue of a linear least-squares fit of the graph in Fig. 1(a). Our residue is about 4 times smaller than that found by Li, Sander, and Meakin[2] (this is presumably the consequence of our averaging method over 50 random positions of the box-counting grid), but it displays a similar concave behavior which is directly associated with the drift observed in the local dimension measurement. At small scales, the local dimension naturally decreases towards 1, the dimension of a line, before ultimately increasing towards 2, the dimension of a surface, since each particle was identified with an elementary cell of our lattice. (This trick was used to soften the crossover to 1.) At large scales, the largest ϵ considered in our box-counting algorithm is of the size of the cluster itself which explains the tendency of the local dimension to increase towards 2.

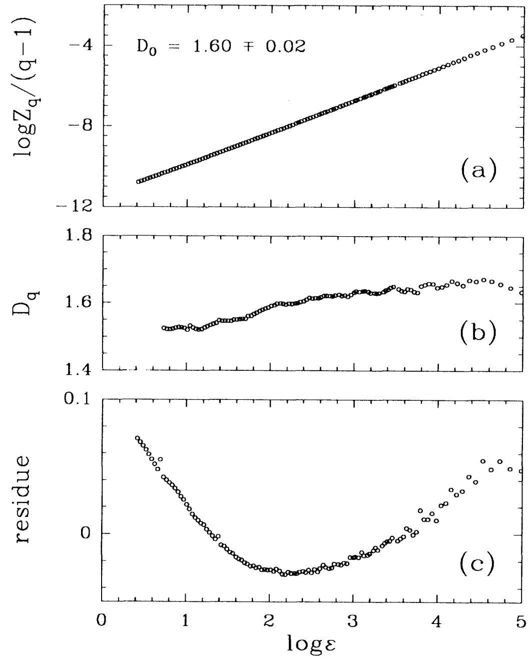

FIG. 1. Box-counting computation of $D_{q=0}$ for an on-lattice DLA cluster of mass $M = 5 \times 10^4$. (a) $\log Z_q/(q-1)$ vs $\log\epsilon$. (b) The local dimension given by the slope of the curve in (a) obtained from linear regression fits for the range $\Delta\log\epsilon = \log\sqrt{2}$. (c) Residue of the linear fit of the graph in (a).

Li, Sander, and Meakin[2] introduced a nonlinear correction term in the linear least-squares fit of $\log Z_q$ vs $\log\epsilon$. There is no perturbative justification for this phenomenological nonlinear term except that it reduces and flattens the residue and yields a new estimate of $D_q = 1.69 \pm 0.03$, in agreement with the dynamical dimension value $D_d = 1.71 \pm 0.02$. This result is somewhat puzzling since this nonlinear estimate does not lie inside the errors bars of the linear least-squares fit $D_q = 1.57 \pm 0.05$ which is quite consistent with our estimate for on-lattice clusters. Moreover, this nonlinear estimate is significantly larger than our local estimate of D_q shown in Fig. 1(b) over the entire accessible ϵ range.

The main feature that seems to be common to both off-lattice and on-lattice DLA clusters is that finite-size effects affect the entire ϵ range accessible, even for clusters of mass $M \sim 10^5$. This can be seen in Fig. 1(b) where the local dimensions D_q do not display any significant plateau for intermediate ϵ values. This is also evident in Fig. 1(c), where the residue in the linear least-squares fit exhibits a rather sharp minimum instead of a flat minimum over a wide range of scales. By the reasoning of Li, Sander, and Meakin,[2] one expects the residue curve to flatten for large cluster mass, but we have not found such a tendency in the range of mass ($M = 10^3$ to 10^5) that we have investigated with on-lattice DLA clusters. The size of the DLA clusters needed to decide whether or not $D_q = 1.60 \pm 0.02$ is an asymptotic estimate may be far beyond the power of current computers. Thus the possibility that the fractal dimension D_0 converges asymptotically to the dynamical dimension value $D_d = 1.71 \pm 0.02$ in the limit of very large clusters is still an exciting numerical challenge.

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