PHYSICAL REVIEW A **69**, 014304 (2004)

Preparation of polarization-entangled mixed states of two photons

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(Received 30 August 2003; published 16 January 2004)

We propose a scheme for preparing arbitrary two-photons polarization-entangled mixed states via controlled location decoherence. The scheme uses only linear optical devices and single-mode optical fibers, and may be feasible in experiment within current optical technology.

DOI: 10.1103/PhysRevA.69.014304 PACS number(s): 03.67.−a, 42.25.Ja, 03.65.Ud, 89.70.+c

Entanglement has played a crucial role for many applications of quantum information, such as quantum teleportation [1], superdense coding [2], quantum error correction [3], etc. To function optimally these applications require maximal pure entanglement. However, unwanted coupling to the environment causes decoherence of quantum systems and yields mixed state entanglement. Therefore entanglement concentration [4–8] and various applications of mixed state entanglement [9–11] are very important and have been investigated by many authors. Experimentally, a special mixed state, "decoherence-free subspace" [12], has been demonstrated and optical Werner states have been prepared [13].

In this paper, we propose a scheme for preparing arbitrary polarization-entangled mixed states of two photons via controlled decoherence. In the experiment of demonstrating decoherence-free subspace, decoherence was imposed by coupling polarization modes to frequency modes of photons. Here we introduce decoherence by entangling polarization modes with location modes of photons, where location modes are finally traced out by mixing them with appropriate path-length differences and detecting coincidences independent of the emission of photon pair from photon source.

Consider a two-qubit mixed state ρ of quantum system AB. It can be represented as [14]

$$\rho=\sum_{i=1}^{4} p_i|\psi_i\rangle_{AB}\langle\psi_i|, \tag{1}$$

where $0\leqslant p_i\leqslant 1$, $\Sigma_{i=1}^4 p_i=1$, and $p_i\geqslant p_j$ for $i\leqslant j$. $|\psi_i\rangle_{AB}$ are two-qubit pure states with the same entanglement of formation as ρ. Therefore $|\psi_i\rangle$ are the same up to some local unitary operations [15], i.e., $|\psi_i\rangle=U_i\otimes V_i|\Phi\rangle$, where U_i and V_i are local unitary operations and $|\Phi\rangle=\cos\theta|00\rangle+\sin\theta|11\rangle$ with $0\leqslant\theta\leqslant\pi/4$.

The experimental arrangement for our scheme is described in Fig. 1. First, spontaneous parametric down-conversion in two adjacent β-barium borate (BBO) crystals produces initial two-photons polarization-entangled pure state $|\Phi\rangle_{AB}=\cos\theta|HH\rangle+\sin\theta|VV\rangle$ [16,17], where $|H\rangle$ and $|V\rangle$ are horizontal and vertical polarizations, respectively. Then six beam splitters with variable transmission coefficients (VBS) couple the initial polarization state $|\Phi\rangle_{AB}$ to location modes and each photon has four possible optical paths $i_{A(B)}$. At each path, single-qubit polarization rotations (SPR) perform local unitary operations U_i,V_i on the polarization mode of each photon and transform the initial entangled state $|\Phi\rangle_{AB}$ to different $|\psi_i\rangle_{AB}$. The four paths of each photon mix on couplers $G_{A(B)}$ and become one through single-mode optical fibres (SMOF) [18]. In experiment, lossless mixture process can be realized by using a fast switch to combine different modes, in addition to a pulsed pump laser. However, this method is not very practical although it can be implemented in principle. A practical replacement can be a passive coupler without switch although it will decrease the optimal success probability due to losses in the coupling. Denote $L_i^{A(B)}$ as the optical path lengths of paths $i_{A(B)}$ (from the BBO crystal to coupler $G_{A(B)}$). If $L_i^{A(B)}$ satisfy $L_i^A=L_i^B$ and $\Delta_{ij}^{A(B)}=|L_i^{A(B)}-L_j^{A(B)}|\geqslant l_{coh}$ for different i and j, the location modes will be traced out and we obtain a mixed state, where l_{coh} is the single-photon coherence length.

The mixed state still contains information of the location modes since photons from different paths i will arrive at the detector at different time. Therefore the mixed state is composed by two discrete subspace state: two photons A, B arriving at same time and at different time. If the time window of the coincidence counter is small enough, only photons from paths with same lengths (i_A and i_B) contribute the counts and the polarization states of photons are reduced to the subspace state with same arrival time. In this subspace, the state is just the two-qubit mixed state ρ. Therefore the final SPR in each arm, along with the polarization beam splitter (PBS), enable analysis of the polarization correlations in any basis, allowing tomographic reconstruction of the density matrix [4,12,17].

In our scheme, we require that the coherence length of pump laser is much smaller than path-length difference Δ_{ij}

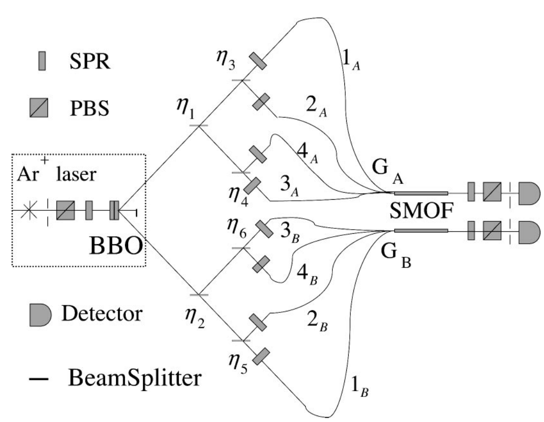

FIG. 1. Experimental setup for preparing an arbitrary polarization-entangled mixed state of two photons.

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in order to avoid two-photons interference. In Ref. [18], two-photons interference can be used to realize a Franson-type test of Bell inequalities, where the path-length difference is two orders of magnitude smaller than the coherence length of the pump laser. In our scheme, we require that the path-length difference is larger than both single photon and pump laser coherence lengths to avoid both single- and two-photons interference. In this case, the travel time of a photon pair from laser to detector enables to resolve the different paths in principle. It is therefore important to postselect only photons arriving in coincidence but not to resolve the travel time from emission to detection in order to trace out all location modes.

The VBS in the scheme can be implemented using a one-order Mach-Zehnder interferometer [19,20] and it transforms location modes in the following way:

$$|a\rangle_{initial} \to \sqrt{\eta_i}|a\rangle_{final} + \sqrt{1-\eta_i}|b\rangle_{final}, \tag{2}$$

where $|a\rangle$ and $|b\rangle$ are the location modes and $\sqrt{\eta_i}$ are the transmission coefficients. The $\mathrm{SPR}|_i^{A(B)}$ at paths $i_{A(B)}$ can be constructed with wave plate sequences [20] and they perform unitary operation $U_i^A(V_i^B)$. The coupler $G_{A(B)}$ introduces decoherence, which yields mixed state

$$\rho_0 = \sum_{i,j=1}^{4} p_{ij} U_i \otimes V_j |\Phi\rangle\langle\Phi| U_i^\dagger \otimes V_j^\dagger, \tag{3}$$

where p_{ij} is the combined probability with photons A and B at paths $|i\rangle_A$, $|j\rangle_B$, respectively. If the time window of the coincidence counter T satisfies $T < \Delta_{ij}/c$ (c is the velocity of the light), only photons with location modes $|i\rangle_A|i\rangle_B$ are registered by the coincidence counter, and density matrix ρ_0 is reduced to

$$\rho = \frac{1}{F}\sum_{i=1}^{4} p_{ii}|\psi_i\rangle\langle\psi_i|, \tag{4}$$

where $F = \Sigma_{i=1}^4 p_{ii}$ is the successful probability of generating the mixed state ρ.

Denote $p_i = p_{ii}/F$. The remaining problem is to find the optimal successful probability F. For a given density matrix of a mixed state, there are several choices of $|\psi_i\rangle$ (to be realized using different local operation and different initial entangled states) and thus also different beamsplitter settings. The best choice is the one where the success probability is maximized. From Fig. 1, we find $p_{11} = \eta_1\eta_2\eta_3\eta_5$, $p_{22} = \eta_1\eta_2(1-\eta_3)(1-\eta_5)$, $p_{33} = (1-\eta_1)(1-\eta_2)\eta_4\eta_6$, and $p_{44} = (1-\eta_1)(1-\eta_2)(1-\eta_4)(1-\eta_6)$. Assume $p_1 \geqslant p_2 \geqslant p_3 \geqslant p_4$, $p_1 > 0$, and $A_i = p_i/p_1$, the optimal values of F can be obtained using Lagrangian multipliers and the results are classified as follows:

1. If $A_i > 0$ for $i = 2,3$, then $\eta_1 = \eta_2 = (1+\sqrt{A_2})/(\Sigma_{i=1}^4\sqrt{A_i})$, $\eta_3 = \eta_5 = 1/(1+\sqrt{A_2})$, $\eta_4 = \eta_6 = \sqrt{A_3}/(\sqrt{A_3}+\sqrt{A_4})$, and $F_{optimal} = (\Sigma_{i=1}^4 A_i)/(\Sigma_{i=1}^4\sqrt{A_i})^2$.

2. If $A_2 > 0$, $A_3 = A_4 = 0$, then $\eta_1 = \eta_2 = \eta_4 = \eta_6 = 1$, $\eta_3 = \eta_5 = 1/(1+\sqrt{A_2})$, and $F_{optimal} = (1+A_2)/(1+\sqrt{A_2})^2$.

3. If $A_2 = A_3 = A_4 = 0$, then $\eta_i = 1$, and $F_{optimal} = 1$.

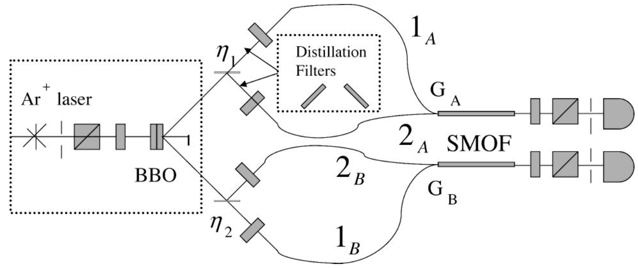

FIG. 2. Experimental set-up used to generate mixed states $\rho = p|\psi\rangle\langle\psi| + (1-p)|\phi\rangle\langle\phi|$.

So far we have described a scheme for preparing an arbitrary polarization-entangled mixed state of two photons using variable beam splitters and single-mode optical fibres. In practical quantum-information process, we often use a special set of mixed states with the form $\rho = p|\psi\rangle\langle\psi| + (1-p)|\phi\rangle\langle\phi|$, where $0 \leqslant p \leqslant 1$ and $|\psi\rangle$, $|\phi\rangle$ are arbitrary two-qubit pure states. Our scheme can be simplified for this special mixed state. Assume $|\psi\rangle = U_1 \otimes V_1|\Phi(\alpha)\rangle$, $|\phi\rangle = U_2 \otimes V_2|\Phi(\beta)\rangle$ with $|\Phi(\theta)\rangle = (\cos\theta|HH\rangle + \sin\theta|VV\rangle)$, and $0 \leqslant \beta \leqslant \alpha \leqslant \pi/4$, the experimental arrangement may be described by the schematic in Fig. 2.

In Fig. 2, VBS and SMOF perform the same operations as those in Fig. 1. The distillation filters are used for entanglement transformation [4] that is performed on location $|1\rangle_A$ or $|2\rangle_A$, depending on the initial state [$|1\rangle_A$ corresponds to transformation $|\Phi(\beta)\rangle \to |\Phi(\alpha)\rangle$ and $|2\rangle_A$ to $|\Phi(\alpha)\rangle \to |\Phi(\beta)\rangle$]. The decoherence process yields different final states for different initial states $|\Phi(\beta)\rangle$ and $|\Phi(\alpha)\rangle$,

$$\rho = \frac{1}{P}[k_1\eta_1\eta_2|\psi\rangle\langle\psi| + (1-\eta_1)(1-\eta_2)|\phi\rangle\langle\phi|],$$

$$\rho' = \frac{1}{P'}[\eta_1\eta_2|\psi\rangle\langle\psi| + k_2(1-\eta_1)(1-\eta_2)|\phi\rangle\langle\phi|], \tag{5}$$

where $k_1 = \sin^2\beta/\sin^2\alpha$ ($k_2 = \cos^2\alpha/\cos^2\beta$) [20,21] is the maximally feasible transformation probability in experiment from $|\Phi(\beta)\rangle$ to $|\Phi(\alpha)\rangle$ [from $|\Phi(\alpha)\rangle$ to $|\Phi(\beta)\rangle$], $P = k_1\eta_1\eta_2 + (1-\eta_1)(1-\eta_2)$ and $P' = \eta_1\eta_2 + k_2(1-\eta_1)(1-\eta_2)$ are the successful probabilities to obtain ρ and ρ'. The transmission coefficients $\sqrt{\eta_1}$ and $\sqrt{\eta_2}$ satisfy condi-

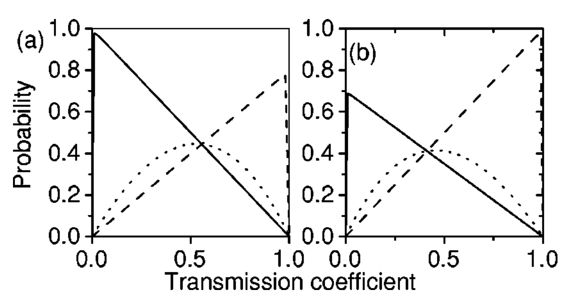

FIG. 3. The final successful probabilities P (a) , P' (b) vs transmission coefficient η_1. $A = 10\,000$ (solid), $A = 1$ (dot), $A = 0.0001$ (dash). (a) $k_1 = 0.8$; (b) $k_2 = 0.7$.

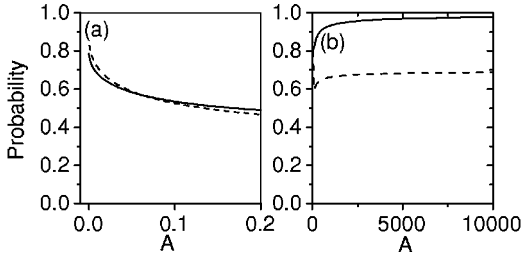

FIG. 4. The final successful probabilities P (soild), P' (dash) vs the ratio A. $k_1=0.8$, $k_2=0.7$. (a) $0\leqslant A\leqslant 0.2$; (b) $0\leqslant A\leqslant 10\,000$.

tions $(1-\eta_1)(1-\eta_2)=Ak_1\eta_1\eta_2$ for ρ and $k_2(1-\eta_1)(1-\eta_2)=A\eta_1\eta_2$ for ρ', where $A=(1-p)/p$. The optimization of P and P' yields that

$$P=k_1(1+A)/(1+\sqrt{Ak_1})^2,$$

$$P'=k_2(1+A)/(\sqrt{k_2}+\sqrt{A})^2, \tag{6}$$

with $\eta_1=\eta_2=1/(1+\sqrt{Ak_1})$ for ρ and $\eta_1=\eta_2=\sqrt{k_2}/(\sqrt{k_2}+\sqrt{A})$ for ρ'. Direct comparison between P and P' shows that if

$$0\leqslant p\leqslant \frac{k_1(1-\sqrt{k_2})^2}{k_1(1-\sqrt{k_2})^2+k_2(1-\sqrt{k_1})^2},$$

then $P'\leqslant P$ and $|\Phi(\beta)\rangle$ is chosen as initial state. Otherwise $P\leqslant P'$ and $|\Phi(\alpha)\rangle$ is used.

In Fig. 3, we plot the successful probabilities P, P' with respect to the transmission coefficient η_1. The probability P (P') reaches the maximum at certain η_1, as predicted by Eq. (6). We notice that there exist fixed points $(\eta_1,P)=(1/(1+k_1),k_1/(1+k_1))$ and $(\eta_1,P')=(k_2/(1+k_2),k_2/(1+k_2))$ for arbitrary parameter A. If the transmission coefficient η_1 is selected at those points, the final successful probabilities P, P' are constants, independent of the ratio of two components $|\psi\rangle$ and $|\phi\rangle$.

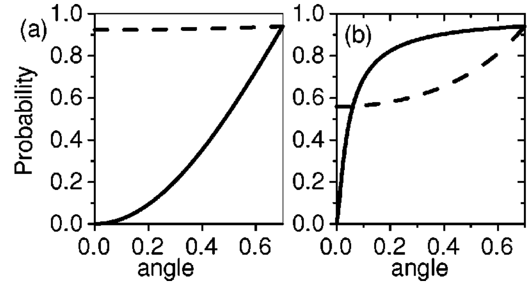

FIG. 5. The final successful probabilities P (solid), P' (dash) vs the angle β. $\alpha=0.7$. (a) $A=0.001$; (b) $A=1000$.

In Fig. 4, we plot the optimal probabilities P, P' with respect to the parameters A. As predicted by Eq. (6), $P\rightarrow k_1$, $P'\rightarrow 1$ as $A\rightarrow 0$; while $P\rightarrow 1$, $P'\rightarrow k_2$ as $A\rightarrow\infty$. These two cases correspond to pure final state ρ. We also notice that there exist minimum values of P (P') at $A=k_1(1/k_2)$.

Figure 5 shows the change of success probability $P(P')$ as β increases from 0 to α. If $A\leqslant 1$, P is always less than P' for any β and initial state $|\Phi(\alpha)\rangle$ is always used; while for $A>1$, $|\Phi(\beta)\rangle$ may be used for large β.

In conclusion, we have described an experimental scheme for producing an arbitrary polarization-entangled mixed state of two photons via controlled location decoherence. The scheme uses only linear optical devices and single-mode optical fibers and may be feasible within current optical technology. We believe the scheme may provide a useful mixed state entanglement source in the exploration of various quantum-information processing.

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