Entanglement Concentration of Individual Photon Pairs via Linear Optical Logic

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We propose a scheme for concentrating nonmaximally pure and mixed polarization-entangled state of individual photon pairs. The scheme uses only simple linear optical elements and may be feasible within current optical technology.

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Entanglement has played a key role in many quantum information processing, such as quantum computation [1], quantum teleportation [2], quantum dense coding [3], and entanglement-assisted quantum cryptography [4]. To function optimally these applications requires maximally entanglement. However, unwanted coupling to the environment causes the degradation of entanglement and entanglement concentration [5] is thus essential for quantum computing. The basic idea of entanglement concentration is to distill some pairs of particles in highly entangled states from less entangled states using local quantum operations and two-way classical communications (LOCC) [1]. In practice, there are two fundamentally different types of concentration protocols: those acting on individual pairs of entangled particles [6] and those acting collectively on many pairs [7]. In recent years, entanglement concentration using linear optical elements has received much attention [8, 9, 10]. In the case of concentration of individual entanglement photon pairs, Thew and Munro proposed a protocol based on beam splitters with variable transmission coefficients (VBS) [9] and experimentally, Kwiat *et. al.* has implemented individual entanglement concentration using partial polarizers [10]. However, both Thew and Munro's protocol and Kwiat's experiment included parameters that are difficult to adjust in practice: Thew and Munro's scheme requires four VBS and the partial polarizers in Kwiat's experiment must be changed according to the initial entanglement.

In this paper, we propose a scheme for concentrating nonmaximally pure and mixed polarization-entangled state of individual photon pairs using linear optical elements (polarization beam splitter (PBS), half wave plate (HWP), and quarter wave plate (QWP)) . The scheme uses only Mach-Zehnder interferometers and a few adjustable polarization rotations (generated by HWP and QWP) and maybe greatly simplify the experiment.

The crucial part in any individual pairs' entanglement concentration scheme is the realization of single-qubit local quantum operator, including unitary rotation and the positive-operator-valued measurement (POVM) [11]. Generally, a single-qubit unitary rotation on the polarization of a photon (or single-qubit polarization rotation (SPR)) has the form $U=\begin{pmatrix} e^{-i\xi}\cos\theta & e^{-i\iota}\sin\theta \\ e^{i\iota}\sin\theta & -e^{i\xi}\cos\theta \end{pmatrix}$ and can be implemented using a wave plate sequence as shown in Fig. 1a, where two phase shifters provide the phase factors $e^{-i\xi}$ and $e^{i\iota}$ and one HWP gives the rotation [12]. Consider a single qubit POVM M_i $(i=1,2)$ satisfying $M_1^\dagger M_1 + M_2^\dagger M_2 = I_2$. They can be represented as $M_1 = \mathrm{diag}(\cos\theta, \cos\vartheta)$, $M_2 = \mathrm{diag}(\sin\theta, \sin\vartheta)$ up to some unitary operations, where I_2 is the unit operation [13]. By using location of each photon as assistant qubit, M_i can be replaced by a two-qubit unitary operator

$$U=\begin{pmatrix} R_y(-2\theta) & 0 \\ 0 & R_y(-2\vartheta) \end{pmatrix} \qquad (1)$$

acting on both polarization and location, where we have used basis $\{|0\rangle_P|0\rangle_L, |0\rangle_P|1\rangle_L, |1\rangle_P|0\rangle_L, |1\rangle_P|1\rangle_L\}$, $R_y(\theta)$ is a rotation by θ around $\hat{y}$, $|i\rangle_P$ and $|i\rangle_L$ represent the polarization and location, respectively. It is easy to testify that U has the decomposition

$$U = V_1 V_2 V_3 V_2 V_1,$$

where V_1 is a location controlling polarization NOT gate (by adding σ_x on location $|1\rangle_L$), V_2 is a polarization controlling location NOT gate (by a PBS), and V_3 represents a location controlling polarization unitary rotation that performs polarization rotation $R_y(-2\theta)$ $(R_y(2\vartheta))$ if the location is $|0\rangle_L$ $(|1\rangle_L)$. With the decomposition of U, a POVM on the polarization of a photon can be realized using a Mach-Zehnder interferometer as shown in Fig. 1b.

With the linear optics realization of the single qubit local quantum operator, we can readily study the entanglement concentration of individual photon pairs. Consider the initial state is an entanglement shared by two spatially separated subsystem, A and B. The qubits used here are polarization states of the photon with $|H\rangle$ (Horizontal), $|V\rangle$ (Vertical) corresponding to the $|0\rangle_P$, $|1\rangle_P$ states above. The experimental arrangement is described by the schematic plot in Fig. 2. The left part, including the BBO crystal and quartz decoherers provides the source of the initial polarization entangled pure [14, 15] or mixed [16] states. The entangled photon pair is then incident to the

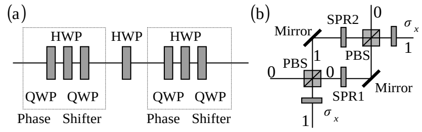

Figure 1: Linear optical realizations of arbitrary local single qubit quantum operator. (a) Single qubit polarization rotation (SPR). (b) Single-qubit POVM. SPR1 and SPR2 perform operation $R_y(-2\theta)$ and $R_y(2\vartheta)$, respectively.

concentration part. By varying the single-qubit polarization rotation (SPR), it can perform arbitrary local single-qubit quantum operation and realize the entanglement concentration. With the prior knowledge of the initial and final states, SPR $A(B)i$ can be determined and adjusted. The final SPR in each arm, along with PBS, enable analysis of the polarization corrections in any basis, allowing tomographic reconstruction of the density matrix [10, 14, 15, 16]. In practice, the tomographic measurements are only performed on the paths with successful entanglement concentration. In the experiment to concentrate entangled pure states, only one Mach-Zehnder interferometer is sufficient.

We first consider the behavior of pure states under the protocol. The concentration is from the partially entangled pure state $|\Phi\rangle = \cos\alpha |HH\rangle + \sin\alpha |VV\rangle$ to $|\Psi\rangle = \cos\beta |HH\rangle + \sin\beta |VV\rangle$, where angles $\alpha, \beta \in [0, \pi/4]$ and $\alpha < \beta$. The concentration can be implemented by a POVM $M_1 = \mathrm{diag}(\cos\omega, 1)$ with optimal successful probability [6]

$$P = \frac{\sin^2\alpha}{\sin^2\beta}, \tag{2}$$

where $\omega = \arccos(\tan\alpha/\tan\beta)$. This POVM corresponds to the unitary operation $U = \mathrm{diag}(R_y(-2\omega), I_2)$ on photon A with post-selecting location $|0\rangle_L$ as the successful output. In Fig. 2, we set SPR $A2$ to perform rotation $R_y(-2\omega)$ and all others are the unit operation. For example, the only operation for concentrating initial entanglement $|\Phi\rangle = \frac{\sqrt{3}}{2}|HH\rangle + \frac{1}{2}|VV\rangle$ to maximal $|\Phi\rangle = \frac{\sqrt{2}}{2}|HH\rangle + \frac{\sqrt{2}}{2}|VV\rangle$ is the adjustment of SPR $A2$ to perform polarization rotation $R_y\left(-2\arccos\left(\frac{1}{\sqrt{3}}\right)\right)$.

Now we turn our attention to the concentration of mixed states. Generally, an arbitrary bipartite density matrix can be represented as [17] $\rho = \left(\sum_{i,j} R_{ij}\sigma_i \otimes \sigma_j\right)/4$, where the summation extends from 0 to 3 with σ_0 the 2×2 identity matrix and σ_1, σ_2, σ_3 the Pauli spin matrices, R_{ij} are real and linear parameterizations. Optimal concentration protocol can be obtained in two cases. If the matrix $R = [R_{ij}]$ is diagonalizable by proper orthotropous Lorentz transformations (POLT), a Bell diagonal mixed state can be extracted with maximal possible entanglement of formation from the initial mixed state with nonzero probability [7, 18]. If R is not diagonalizable by POLT, the probability of obtaining Bell-diagonal state is equal to zero. However, it can still be quasi-distillated [18, 19]. Optimal local quantum operation can be calculated explicitly according to the POLT [18]. For both cases, the local quantum operator can be written in the form

$$U_A \begin{pmatrix} \cos\theta_A & 0 \\ 0 & \cos\delta_A \end{pmatrix} U'_A \otimes U_B \begin{pmatrix} \cos\theta_B & 0 \\ 0 & \cos\delta_B \end{pmatrix} U'_B. \tag{3}$$

The optimum is in the sense that we can choose suitable parameterizations $U_{A(B)}$, $U'_{A(B)}$, $\theta_{A(B)}$, $\delta_{A(B)}$ to realize optimal entanglement concentration. Similar as those shown for entangled pure states, we can perform the single qubit unitary operations $U^{(\prime)}_{A(B)}$ and POVM by varying the SPR $A(B)i$, therefore our linear optical protocol can implement entanglement concentration for an arbitrary initial entangled mixed state.

It is interesting to compare our protocol with Thew and Murno's, which used four VBS to obtain an effective transmission matrix $A \otimes B = \mathrm{diag}(\eta_{HA}\eta_{HB}, \eta_{HA}\eta_{VB}, \eta_{VA}\eta_{HB}, \eta_{VA}\eta_{VB})$. In their protocol, Thew and Munro asked the question why there are four individually tunable filters η_{HA}, η_{HB}, η_{VA}, η_{VB}. As we can see from the local quantum operator in (3), four individually tunable filters are the minimum requirement for implementing arbitrary local quantum operations! Obviously the effective transmission matrix in Thew and Munro's scheme can be obtained by setting $\cos\theta_{A(B)} = \eta_{HA(B)}$ and $\cos\delta_{A(B)} = \eta_{VA(B)}$, therefore all Thew and Munro's discussion about entanglement concentration can be applied to ours. However, our protocol is more feasible within current linear optical technology because it need only Mach-Zehnder interferometer and some HWP and QWP.

In conclusion, we have proposed an experimentally feasible protocol for implementing arbitrary local single-qubit quantum operations on individual polarization-entangled photon pairs using linear optical devices (PBS, HWP, QWP). Based on this technology, we have discussed concentration for entangled pure and mixed states with a single copy. We

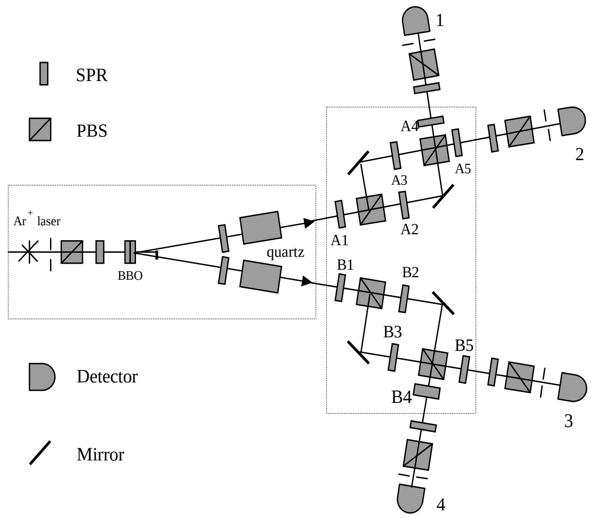

Figure 2: Experimental set-up for entanglement concentration.

emphasis its simplicity and university. For example, it can also be used in multi-partite entanglement manipulation [20]. We believe the scheme should provide a useful tool in the exploration of various quantum information processing.

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