New Turn for Cracks

M. Marder
Department of Physics and Center for Nonlinear Dynamics
The University of Texas at Austin, Austin TX, 78712, USA

Anyone who has had the maddening experience of watching a crack take months to make its way across his car windshield knows that the path left by a fracture can be complicated. At unpredictable points, the crack may decide to wiggle, turn an abrupt corner, or even split into two. Now A. Yuse and M. Sano[1] of Tohoku University have devised an experiment which allows such crack motion to occur in a thoroughly controlled and reproducible setting. The types of patterns they observe, and the manner in which the patterns occur, are remeniscent of phenomena well known in solidification of metals, or in fluid flow, but have not previously been measured for fracture. Furthermore, the pattern-forming process seems to provide a detailed measurement of a fundamental quantity in fracture, the fracture energy, which cannot easily be obtained in other ways.

The experiment of Yuse and Sano (Figure 1) involves pushing a thin glass plate from a hot region to a cold one, so that a temperature gradient travels across it. A crack is introduced by the experimenters, who notch the plate, and as the plate begins to move the crack tip jumps just ahead of the thermal gradient, and remains there. When the strip moves at low velocities, the crack forms a perfect stable cut through the center of the strip. As the velocity increases, however, there is a transition, and the fracture path begins to oscillate, producing the remarkable spectacle of a piece of glass slowly and spontaneously being cut in the shape of a sine wave. Further increases in velocity bring crack branching, and complicated sets of multiple cracks.

A close parallel to their experiment is directional solidification[2] (Figure 1). In directional solidification, a molten solid is pushed at a constant speed into a cold bath, where it hardens. In casting metals it might seem desirable to push as rapidly as possible, but instabilities intervene. As the pushing velocity increases, the interface between solid and liquid undergoes a series of transitions, forming progressively more and more complicated patterns. These structures alter the mechanical properties of metals, and provide a fascinating example of how natural phenomena can create complicate patterns from almost completely homogeneous external conditions.

The reason that cracks propagate in Yuse and Sano's experiment can be understood by thinking about the effect of thermal gradients upon the glass plate. In the hot region, the plate is slightly expanded, while in the cold region it is slightly contracted. Since the hot and cold regions are glued seamlessly together, there must be stresses where they join. The hot thick portion of the strip is pinched where it joins the cold narrow portion, and its effort to spring back from the pinching provides the forces needed for crack propagation. One can think of a crack as an object that sucks stress energy out of a plate the way a vacuum cleaner picks up dust. At any given speed v , the crack requires a certain amount of energy per unit length $\Gamma(v)$ to move, so that if more is provided by the plate, the crack will use it to accelerate.

The solid upper curve in Figure 2 shows the energy per length the strip provides to a straight crack running along its center. The position of the crack is measured with respect to the point at which the strip is cooled; far behind this point, or far ahead of it, the energy available for crack motion vanishes. The height of the curve is proportional to the square of the temperature difference ΔT along the strip. At any given speed v, the tip of the crack will find a spatial location where the energy it derives from this curve equals the energy it needs, $\Gamma(v)$, a location always in front of the temperature gradient, for precisely the same reason that a surfer travels in front of a wave rather than behind it. If the temperature ΔT rises, the solid curve grows in scale, pushing the tip of the crack forward.

The striking feature of Yuse and Sano's experiment is not that the cracks move, but that simple straight line motion becomes unstable in a reproducible way. The condition for instability of a straight crack has been worked out by Cotterell and Rice[3], and is represented by the lower dotted line in Figure 2. When this quantity, a tension parallel to the crack, becomes positive, the slightest imperfections in the plate will be amplified and quickly cause the crack to leave the central axis. So long as the tip of the crack lies within the shaded region of Figure 2, crack motion is stable, but once the temperature difference ΔT becomes too large the crack tip is pushed forward of it, and oscillations begin. These observations explain the basic features of Figure 3 in Yuse and Sano, and show that their experiment can be used as a tool to measure in detail the velocity dependence of $\Gamma(v)$. If $T_{\text{inst.}}(v)$ is the locus of points in their Figure 3 where the first instability occurs, then in the limit where the shape of the temperature gradient is independent of velocity v, one has $\Gamma(v) \sim \sqrt{T_{\text{inst.}}(v)}$. Not only do these lovely experiments bring fracture within the family of problems that may be studied by techniques of pattern formation and nonlinear dynamics[2,4], they should also teach new lessons about the process of fracture.

References

1 Yuse, A., and Sano, M. on page xxx of this issue.

2 Langer, J. S. *Reviews of Modern Physics*, **52** 1-28 (1980).
3 Cotterell, B., and Rice, J. R., *International Journal of Fracture*, **16** 155-169 (1980).
4 Manneville, P., *Dissipative Structures and Weak Turbulence* (Academic Press, Boston, 1990)

Figures

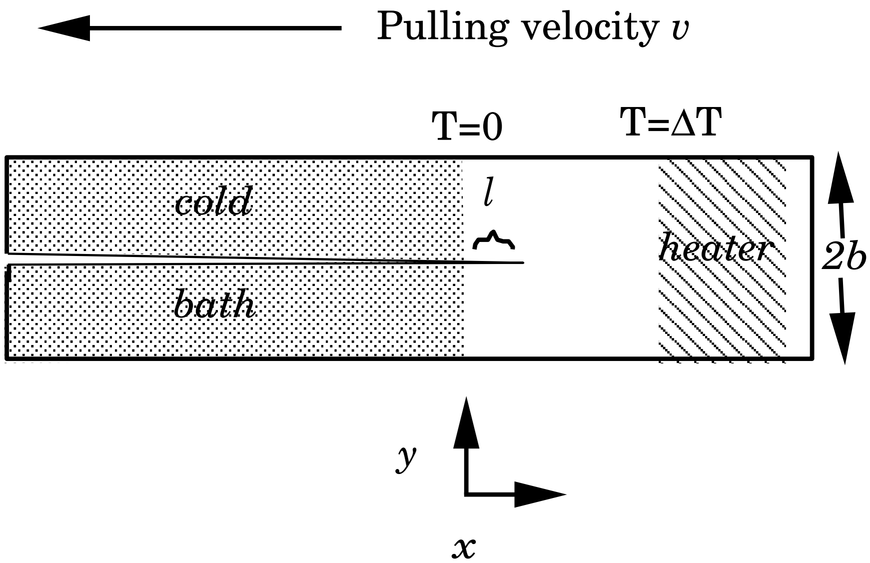

FIGURE 1: On the right is a sketch of directional solidification, and on the left a sketch of the experiment of Yuse and Sano. Both experiments feature strips slid from hot to cold regions, and exhibit a sequence of instabilities as the sliding velocity increases. In each drawing, successive figures in the vertical direction represent steady-state patterns achieved in successive experiments where the constant velocity v increases. In the case of directional solidification, on the right, patterns form at the interface between a hot melted region at the top and a cold solid region below. For slow sliding velocities v, the solidification front is just a straight line, but at some critical velocity, the front begins to ripple. At higher velocities, the ripples become more pronounced, and eventually transform into fingers, which grow in parallel, leaving deep grooves behind them. At higher velocities yet, the fingers also become unstable, and create a complicated branching pattern. Similarly, Yuse and Sano find that at low sliding speeds a straight crack is stable, but it begins to oscillate at a critical speed. At yet higher velocities, the cracks become unstable to complicated branching patterns.

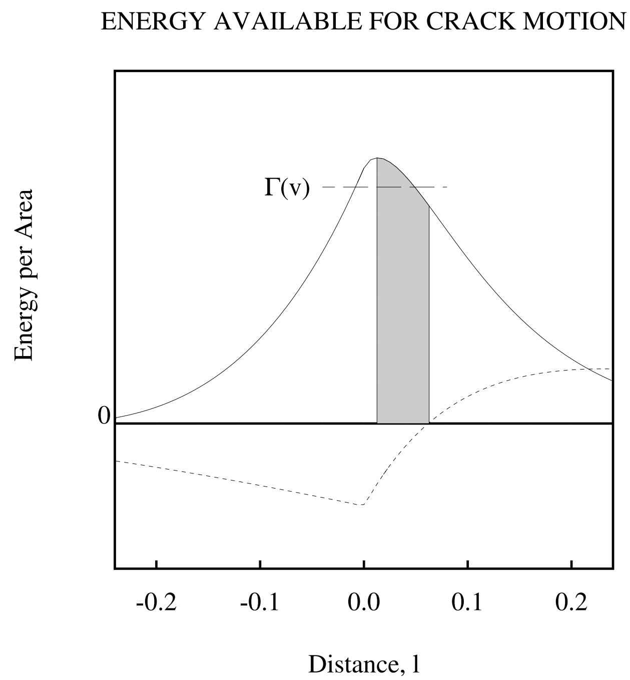

FIGURE 2: The upper solid curve shows the energy available per unit length for crack propagation in the experiment of Yuse and Sano. The horizontal axis shows the position of the crack tip relative to the cold end of the temperature gradient, and is measured in units of the width of the plate. The scale of the vertical axis is not indicated, but the only important fact is that the vertical scale is proportional to the square of the temperature difference ΔT along the strip. The crack will propagate with its tip at a spatial location given by the intersection of the solid curve with the dashed line labeled $\Gamma(v)$. The lower dotted line monitors the stress field responsible for the stability of the crack. When the crack tip reaches the spatial location where this quantity is positive, it becomes unstable; the shaded region indicates the range of stable crack positions.