Stochastic cooling in confined geometries

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The implementation of stochastic cooling in a confined geometry is discussed, where the boundary shape is found to be an independent control parameter for arriving at a variety of final spatial and momentum distributions. Results for integrable and nonintegrable geometries are contrasted in terms of both efficiency of cooling and mechanisms for saturation.

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1. INTRODUCTION

Over the years advances in laser cooling of atoms have resulted in a number of significant scientific results, most notably the formation of atomic condensates.[1] Even at somewhat warmer temperatures than those required for the gas–fluid transition, cold atoms have been used to address a variety of dynamical issues bridging atomic and condensed-matter physics.[2,3] Despite these successes, current techniques in laser cooling have limited applicability to only a small subset of available atoms. As a consequence, there has been an effort in recent years to explore new methods for cooling, both with and without the use of lasers, some of which are discussed in this feature issue.

One of the proposed methods[4] was based on a well-known technique in accelerator physics called stochastic cooling. There the transverse drift that diminishes the brightness of the beam is reduced by a measurement, away from the beam center, followed by a correcting impulse once every beam cycle. In Ref. 4 an analogous procedure was proposed for cooling atoms by use of far-detuned lasers. The essential requirements are a simple feedback scheme to measure either the mean momentum or position of part of a particle (atom) cloud and then a mechanism to apply a force (kick) to reset the center of mass momentum or position of the measured section to zero. After the kick, remixing is necessary to bring a new subset of particles into the kicking region, and the procedure is repeated. It is clear that applying this technique to the entire cloud would negate the value of this iterative scheme.

The basic mechanism can be illustrated by a simple model based on the schematic shown in Fig. 1(a). Consider pairs of numbers (q_i, p_i), $i = 1,\ldots,N$, denoting the positions and momenta of N particles in phase space, chosen at random; $q > R$ denotes the region where the center-of-mass momentum is to be measured and then corrected down to zero. The evolution between kicks is taken in this example to be harmonic, which, in phase space, amounts to a simple rotation of the individual (q_i, p_i), i.e.,

$$p_i' = p_i \cos\alpha - q_i \sin\alpha \tag{1}$$

$$q_i' = p_i \sin\alpha + q_i \cos\alpha, \tag{2}$$

where the angle α can be taken to be fixed or random. The case of fixed α is shown in Fig. 1. The schematic in Fig. 1(a) illustrates the basic process in terms of the motion of the center of mass of the region. Following each measurement, the center-of-mass momentum p_{CM} is kicked to zero. The harmonic evolution between kicks results in a rotation of the center of mass (though the radius is not necessarily constant as in the schematic) till the next measurement and correction. In the idealized case, the process saturates once q_{CM} moves well inside $q = R$. The results of a simulation are shown in Fig. 1(b), and, though the trajectory of the center of mass is more complicated, the basic mechanism is seen to be the same.

Several generic consequences become immediately apparent from this illustration: (i) It is clear that the procedure leads to true compression in phase space as the spread in both q and p is reduced. (ii) The need for effective remixing is also clear, or else the procedure saturates early in the process. Note that remixing essentially ensures that the same particles are not addressed during successive measurement and correction cycles. (iii) Further, the limiting distribution for this particular implementation is a cloud of radius R, where particle accumulation occurs along the circumference. The final distribution of particles shown in Fig. 1(b) illustrates this feature. (iv) Finally, the rate of cooling decreases with time as the center of mass of the section approaches the boundary. Both spatial aggregation of particles and a decrease in cooling efficiency are general features, though the details of the final distribution and the onset depend on the particular realization. Nonlinear evolution between kicks does not necessarily help, as there is quite often heating (expansion in phase space) that overwhelms the cooling per cycle.

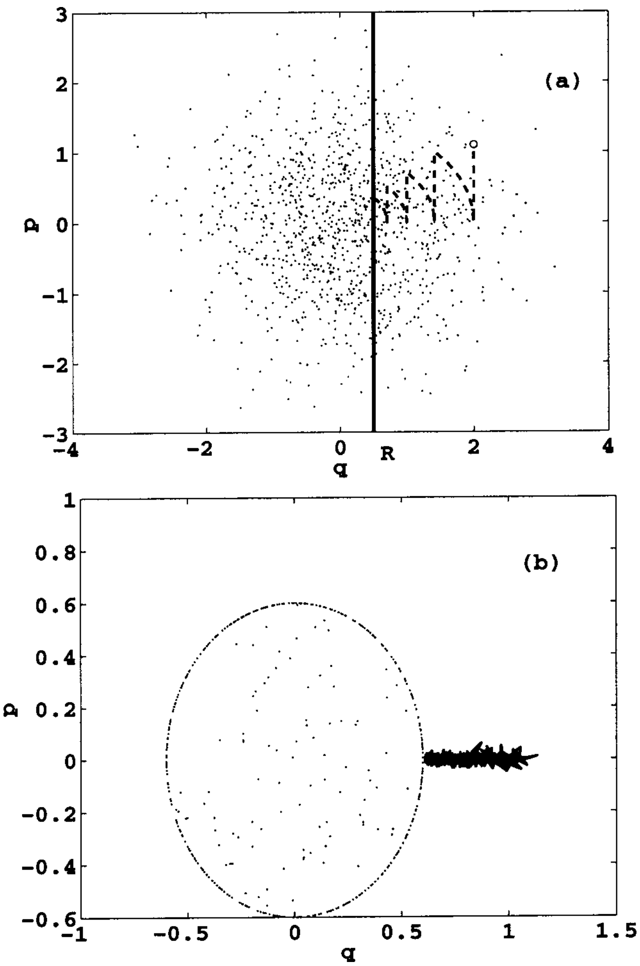

Fig. 1. (a) Schematic of the cooling mechanism for a single kicking regime. The dots represent the particles; the solid vertical line demarcates the spatial measurement and correction area which lies to the right of the line; the dashed curve represents the idealized trajectory of the center of mass of the region. (b) Results of a simulation in which the final distribution of particles in phase space and the actual trajectory of the center of mass of the kicking region (solid curve) are plotted.

We note that two key elements of stochastic cooling also appear in recent suggestions[5,6] for cooling atoms in a cavity. The cavity response to atomic motion accounts for both measurement and kick aspects of the cooling process, though the mixing component was not addressed in either treatment.

An extension of stochastic cooling was proposed[7] that was to use finer spatial resolution, enabling a multiple subdivision of each particle bunch. The resulting sensitivity to higher-order correlations of the distribution can dramatically improve the efficiency of stochastic cooling, though it is harder to implement experimentally. However, the work presented here reflects our view that using the dynamics first to control the nature and size of correlations is perhaps simpler and more efficient, beyond which multiple measurements can be explored.

2. ROLE OF CONFINED GEOMETRIES

We choose here to exploit another recent advance in atom-optics experiments,[8,9] namely, the ability to draw arbitrary (in principle) boundaries around a cooled cloud of atoms. Given the classical regime of the problem, the resulting atomic billiard dynamics is a function of the confining geometry. In brief, it is well known that with a boundary such as a rectangle, square, or circle the motion of a bouncing particle is integrable, which means that the particle trajectories are either periodic or quasi periodic. This is in stark contrast to a stadium geometry, where, with the exception of a few rare, unstable periodic trajectories, the motion is strictly aperiodic and chaotic.[10–11] The aperiodicity ensures that particles visit all parts of the confined region and are thus efficiently mixed. Although it is possible, by altering the boundary, to realize situations in which periodic and aperiodic motions coexist in phase space, these so-called mixed-phase spaces are not as well understood. We note here that mixed-phase spaces are of consequence to any experimental realization of billiard geometries, as the walls of the boundary are soft.[12] This smoothening of the walls can lead to phase-space structures that trap particles, thereby altering the effect of the mixing. However, for the purposes of this paper, we restrict our analysis to the relatively clear-cut cases of integrable circle and nonintegrable stadium geometries. For the stadium, unless otherwise stated, the numerical results shown are for an aspect ratio (ratio of the length of the straight line segment to the radius of the semicircular end caps) of one.

The immediate advantages of implementing stochastic cooling in a confined billiard geometry are that (i) the spatial extent is automatically limited and (ii) the mixing can be controlled by changing the shape of the boundary. Further, a combination of boundary geometry and location of the spatially resolved measurement can be used to control the compression (reduction of the width) either in space or momentum, and in many cases simple geometrical arguments can be used to predict the outcome. For example, Fig. 2 shows three configurations where, in each case, the shaded box represents the region for the measurement and correcting kick. The measurement and kicking is in only one direction, say x. However, the dynamics (bouncing off the walls) mixes p_x and p_y, and cooling is achieved in both directions. Thus the essential transformations in momentum space are rotation interrupted periodically by a shear associated with the kick.

In case (a) of Fig. 2 it can be shown that for any given kicking area there exists a range of angles (with respect to a radial vector) for which particles with angle α satisfying $\pi/2 \geqslant |\alpha| \geqslant \alpha_{\mathrm{cr}}$, where α_{cr} is a critical angle de-

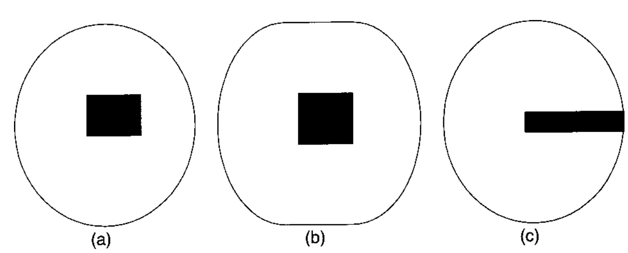

Fig. 2. Configurations of billiard boundaries and measurement regions that lead to differing amounts of cooling and distinctive momentum or spatial distributions. (a), (c), integrable cases; (b) nonintegrable case. In each case the shaded box represents the region for the measurement and correcting kick.

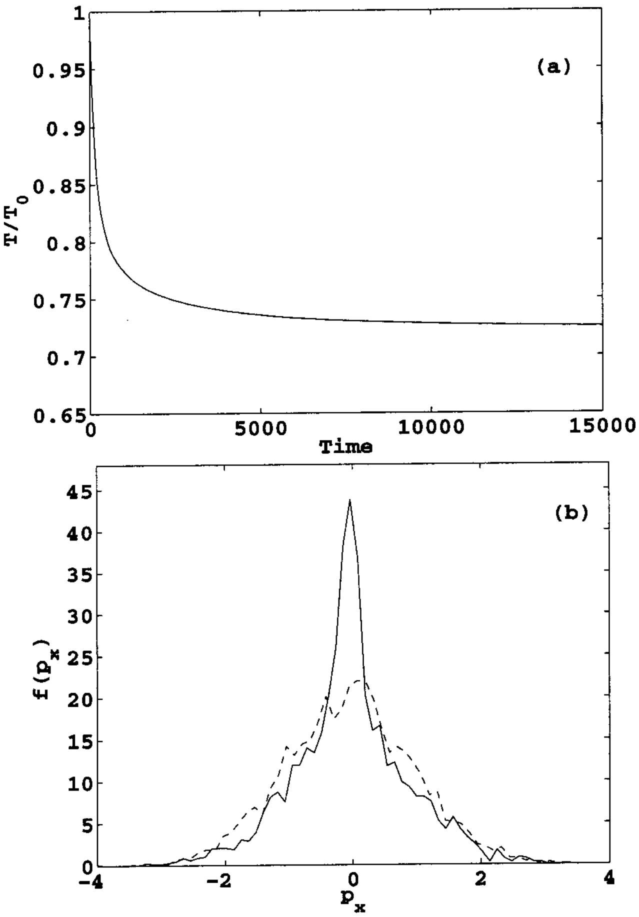

Fig. 3. (a) Effective temperature (see text for definition) ratio for $N = 4000$ particles in a circular billiard, for the kicking location shown in Fig. 2(a). The kicking box is 10% of the total area; time reflects the number of cooling cycles. (b) Initial (dashed) and final (solid) velocity distributions at $T = 10{,}000$ demonstrating the resulting two-temperature distribution.

pending on the specifics of the kicking box geometry, will never be kicked for any momentum. In other words, particles in an annular region are never addressed by the measurement–kick process. The upper panel in Fig. 3 shows the effective temperature, defined as $(p_x^2 + p_y^2)$ averaged over the ensemble, as a function of time. The saturation in this case is due to the exclusion of a subset of particles from the entire cooling process. As a consequence, an initial, thermal, Gaussian velocity distribution evolves into a final distribution consisting of a narrow spike centered at $p_x = p_y = 0$ superposed on a Gaussian background of particles that were never kicked. This effective two-temperature distribution, shown in Fig. 3(b), can be cropped to obtain a cooled sample but, like evaporative cooling, at the cost of reduction in particle number.

The exclusion of a subset of particles does not occur in the case of the stadium billiards shown in Fig. 2(b), and as seen from the dashed curves in Fig. 4 the cooling continues for a longer time and is more efficient than for the circular geometry (solid curve). Case of Fig. 2(c) is an integrable stadium geometry in which the exclusion of particles is no longer an issue, as the kicking region extends from the center to the edge. However, saturation of the cooling occurs here as well for quite different reasons. After being kicked, which in this case means that the average p_x is set to zero, the particles emerge with a velocity closer to vertical from the kicking region. Thus, trajectories spiral out (spatially) toward the stadium perimeter. A trajectory emerging near vertically close to the edge of the circle hits the wall at grazing incidence. This means that it takes a large number of bounces (time) before it reenters the kicking region. Thus the cooling rate slows down as the time between kicks, measured in number of bounces, increases considerably. In this configuration the spatial distribution in the radial direction narrows as the density of particles near the boundary increases with the number of kicks, providing the distinctive high-density annulus of particles shown in Fig. 5. As such, effective compression in terms of the full four-dimensional phase space is significantly higher than in the case of Fig. 2(a).

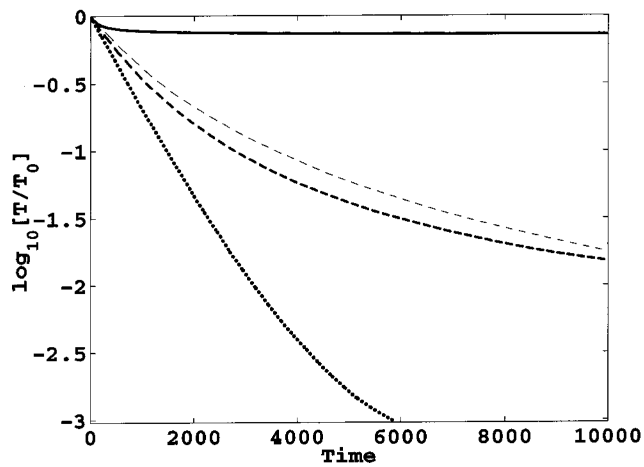

Fig. 4. Behavior of the effective temperature with time for the stadium billiards (light dashed curve); circle geometry (solid curve) for $N = 4000$ and kicking box located similarly to Fig. 2(a) (circle) and Fig. 2(b) (stadium). The case of random kicking-box location is also shown for the circle (heavy dashed curve) and stadium (dotted curve) geometries. In all cases, the box size is 10% of the total area enclosed by the billiards.

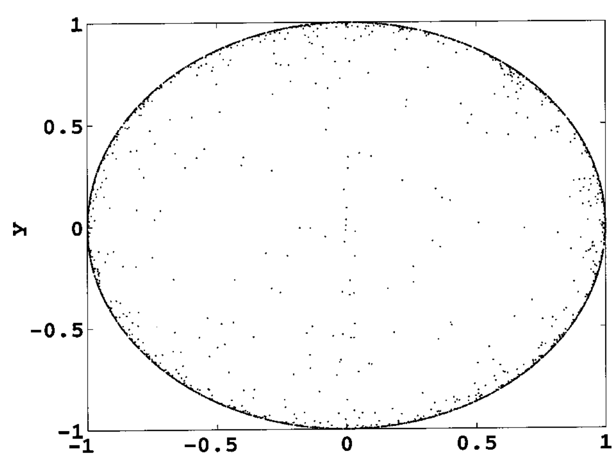

Fig. 5. Final velocity distribution for the geometry in Fig. 2(c). Note the accumulation of particles along the boundary (which is not explicitly shown but is defined here by the line of high density) of the billiards.

Also shown in Fig. 4 are two cases corresponding to randomizing the location of the measurement–kicking box with time. In the case of the circular geometry, the dramatic improvement in cooling rate and extent is expected as the moving location both offsets the effect of the periodic trajectories and improves the sampling of the distribution. What is more surprising is that (i) the circular geometry with random location does better than the stadium, and (ii) randomizing the box location leads to a substantial effect even in the case of the stadium boundary.

Randomizing the box location for the circular boundary cools better than in the stadium with fixed measurement location because of an important class of periodic trajectories in the stadium. These are the so-called bouncing-ball orbits corresponding to normal incidence on the straight-line segments of the stadium. In our case, as the particles are being kicked in the x direction, they emerge (on average) closer to normal incidence. With time, more and more particles satisfy this condition and $\langle p_x \rangle$ in the kicking box becomes smaller, leading to a decrease in cooling efficiency. This continues till a substantial set of particles translates along the straight line segments to the semicircular end caps, beyond which rotations mix p_x and p_y and cooling can resume. The effect is more pronounced for stadium geometries with larger aspect ratios, which is indeed confirmed in our simulations. This understanding also suggests that kicking along the length of the stadium is more beneficial than across the width. Alternatively, as seen in Fig 4, randomizing the location of the measurement also works well to minimize the effects of the bouncing-ball orbits. The use of billiard configurations in which the bouncing-ball orbits are excluded, as in the case of the stadium billiards with tilted straight-line segments,[9,11] would also reduce correlations in the dynamics and help maintain cooling efficiency.

3. CONCLUSION

We have used a numerical study to support the suggestion that implementing stochastic cooling in a confined geometry greatly improves both cooling efficiency and control over the final spatial or momentum distribution of particles. This is significant as, in contrast to its accelerator-physics counterpart, a key limitation to any atomic realization of stochastic cooling is the total number of measurement–kicking cycles. This constraint results not only from heating associated with the measurement, though this is related to the type of measurement, but is also due to fundamental time-scale limitations in atom-optics experiments.[3,8,9]

In an experimental context, it is worth noting an important feature that distinguishes the billiard configuration from the harmonic dynamics considered earlier. As stated earlier, though only one component of the momentum (say p_x) is kicked, the dynamics ensures that energy is extracted from p_y as well because of rotation from the bounces. This aspect results in cooling even in the absence of a spatially resolved measurement, which is not possible with harmonic mixing. The absence of the need for spatially resolved measurements could be very useful for experimental realizations, though there will be a commensurate decrease in cooling rate.

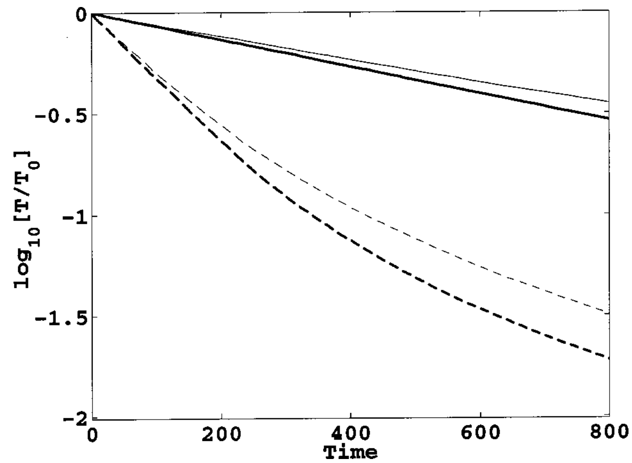

Fig. 6. Behavior of the effective temperature for short times for the stadium billiards with $N = 4000$, contrasting single and multiple (five) spatial measurements. In the multiple-box case, a line of boxes along the x-direction was considered. Solid curves, single box; dashed curves, 5 boxes; for darker curves the kicking is in y.

The key issues for any experimental realization remain the optimization of the cooling rate and, consequently, the ultimate temperature that can be reached over the duration of the experiment. Simple statistical arguments[4] suggest that significant cooling requires that the number of iterations be of the same order of magnitude as the number of particles. As seen from Fig. 6, in a stadium geometry, a single spatially resolved measurement (solid curves) leads to a factor of 2 change over times much shorter than the simple estimate. However, the best strategy is still to make as many spatially resolved measurements as possible. This assertion is supported in Fig. 6, where the dashed curves illustrate the situation in which five spatially resolved measurements are made and where an order-of-magnitude improvement in cooling is seen. Ongoing work aims to provide the theoretical understanding necessary to optimize the cooling rate with respect to geometry and number of measurements, a difficult problem even in an ideal model.

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