GUIDE TO SUPPLEMENTARY MATERIAL – A MATLAB GUI FOR $\langle \boldsymbol{J}_\psi \rangle$

The Matlab code and GUI for the stream function method for determining the energy flux and power by Lee, Paoletti, Swinney, and Morrison[1] are available at [URL will be inserted by AIP]. The latest version is available at the following URLs: `http://www.mathworks.com/matlabcentral/fileexchange/44833` and `http://chaos.utexas.edu/wp-uploads/2013/12/internalwaves_streamfunction_fluxfield.zip`. This guide contains information that is needed to use the GUI. All of the following equation references are from the above-mentioned *Physics of Fluids* paper.

Input Data Format

The user must first supply the .mat file which contains the velocity components, the grid, and a fluid parameters array containing the background density and buoyancy frequency information. The names of the various arrays can be user-specified, but the defaults are as follows. Horizontal velocity: u, vertical velocity: w, horizontal coordinate: x, vertical coordinate: z, fluid parameters: h_rho0_N.

<u>Velocity components</u>: The velocity components must be two separate arrays of identical shape. The first dimension is the z direction, the second dimension is the x direction, and the third is time. The units for the inputs for the program are cgs.

<u>Coordinate arrays</u>: The coordinate arrays must be in the same shape as the velocity components minus the time dimension and must also be separate arrays for the x and z coordinates. The arrays are in the form of outputs for the Matlab function "meshgrid." Refer to the Matlab help documents for further details.

<u>Fluid parameters</u>: The fluid parameter array should contain as its first column the heights at which the background density (second column) and buoyancy frequencies (third column) are evaluated. The heights need not match with the z-component coordinate array specified previously; the values for the background density and buoyancy frequency will be interpolated (cubic) to fit it. If the Boussinesq approximation with uniform reference density and N is being used, the two values can be input as scalars.

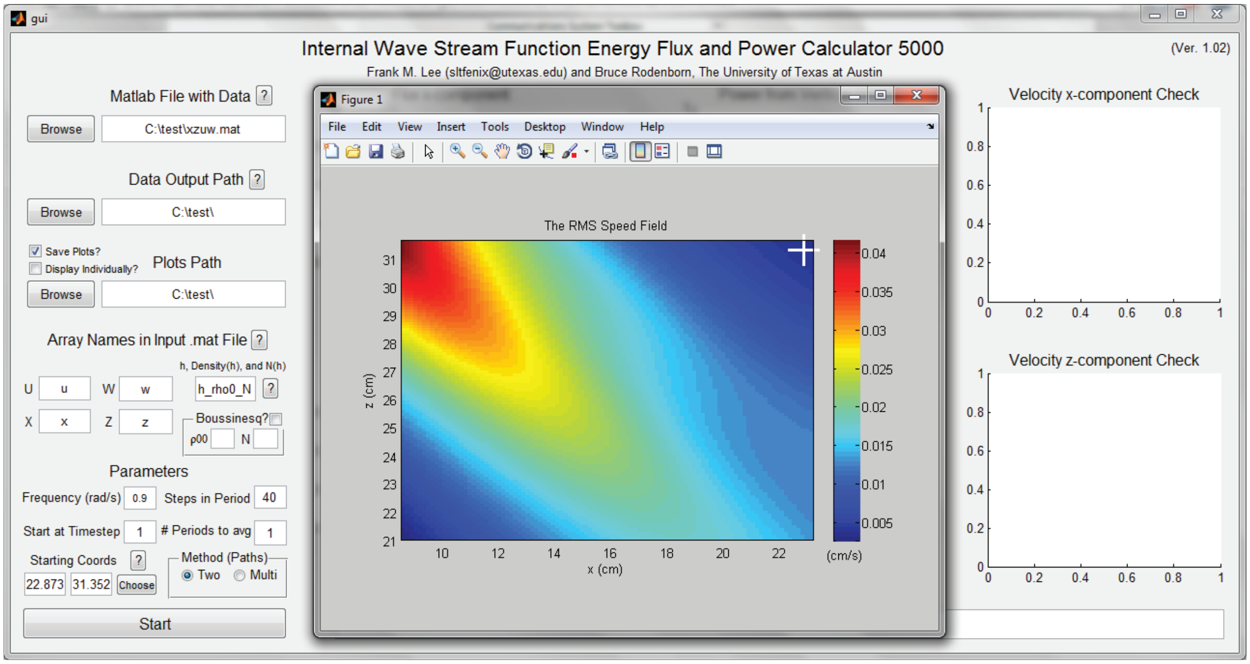

FIG. 1. The user can specify the starting point of the stream function calculation by clicking on a plot of the RMS speed field of the input data.

Other Parameters

The user then specifies the relevant parameters. The frequency of the internal wave field must be supplied in rad/s. Additionally, the number of timesteps in a period of oscillation, the timestep at which to start evaluating the power, and how many periods to average over must be specified. The energy flux expression is time-averaged over an integer number of periods. Additionally, the starting coordinates (in cm) must be specified for the stream function calculation, which can be chosen by clicking on a displayed plot of the RMS speed field, as shown in Fig. 1. The stream function is taken to be zero at those coordinates at all times. The user can also choose between the two-path and multi-path methods. The multi-path method is roughly an order of magnitude slower than the two-path method, and should be used to reduce the error if the data supplied has a lot of noise.

Calculation of the Stream Function

Once all the data and parameters are supplied, the algorithm uses trapezoidal quadrature of the x-velocity values along the z-coordinates, and the z-velocity values along the x-

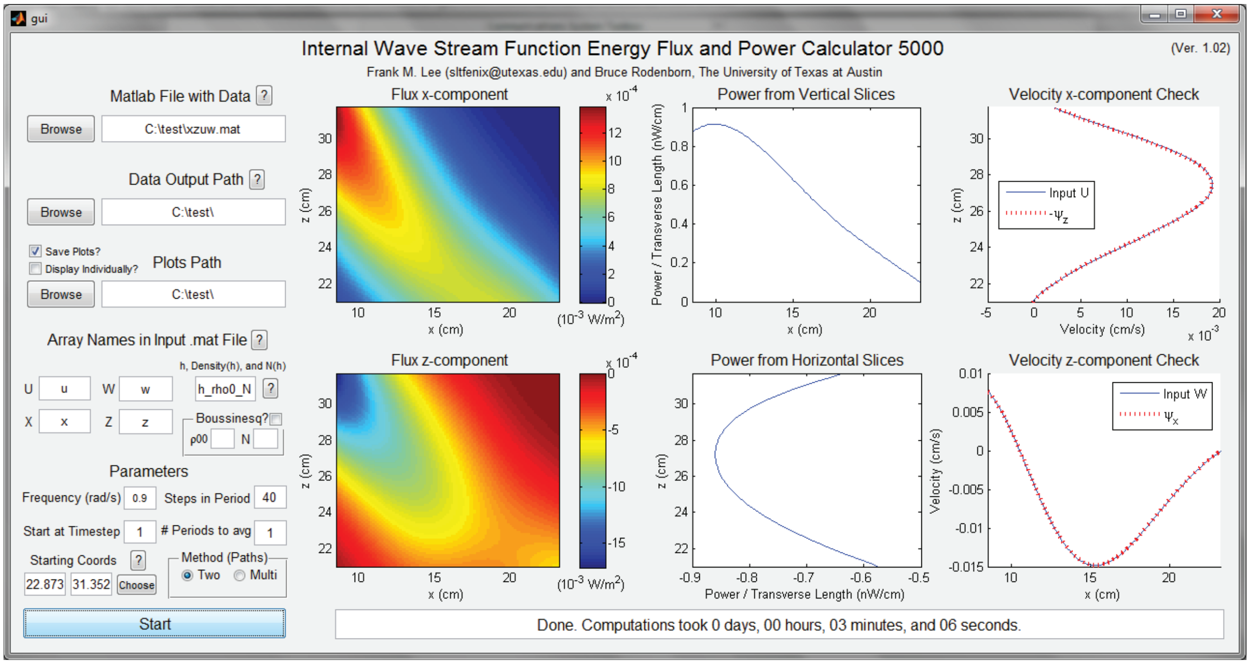

FIG. 2. After the various input parameters are inserted into the GUI, the Matlab program calculates and displays the flux fields, the powers, and velocity checks.

coordinates to find the stream function at each grid point. For the two-path method, it will average over two simple L-shaped paths from the starting point to the evaluation point given by Eqs. (16) and (17). For the multi-path method it will average over every Z-shaped path within the box that forms between the starting point and the evaluation point which is given by Eqs. (18) and (19). Note that if the starting point and evaluation point have the same x or z coordinate, then the only possible path is a straight line. The two-path method calculates only two path integrals for each grid point (excluding the points in line with the starting point), which means it will integrate over $2MN - M - N$ paths, where M is the grid size in x, and N is the grid size in z. The multi-path method calculates $M + N + 2$ paths for each point, where M and N are the number of grid points between the evaluation point and the initial point in the x and z directions. Then the total number of paths integrated for the whole grid is $\frac{1}{2}[M^2N + MN^2 - (M + N)^2 + 3(M + N) - 4]$. The stream function is found for every timestep in the specified range. Derivatives of the calculated stream function are taken and checked against the input velocity components at the initial timestep at the middle of the domain.

Calculation of the Energy Flux

Once the stream function $\psi(x,z,t)$ has been calculated, $\varphi(x,z)$ and its derivatives are calculated (Eqs. (20) – (22)). The real part of φ is found by trapezoidal quadrature in the time direction at each grid point where the integrand is the product of the stream function and $\cos\omega t$. The imaginary part is found using $\sin\omega t$ in place of $\cos\omega t$. The derivatives are done the same way except the velocity components are used instead of the stream function. Then the energy flux (Eq. (13)) is calculated using these quantities. The flux fields and the powers are displayed (Fig. 2) and output into both .txt and .mat files to the specified folder.

REFERENCES

[1]F. M. Lee, M. S. Paoletti, H. L. Swinney, and P. J. Morrison, "Experimental determination of radiated internal wave power without pressure field data," Phys. Fluids , (2014).