REPLY

Reply to comment by McDougall and Barker on "Buoyancy frequency profiles and internal semidiurnal tide turning depths in the oceans"

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1. Introduction

In a recent paper [*King et al.*, 2012], we studied the turning depths of internal tides in the lower layers of the ocean. The turning depth is determined by latitude, wave frequency, and the buoyancy frequency N. Exploring all available WOCE data, we adopted *King et al.* [2012, equations (2) and (3)] for the calculation of N^2. These are the same as equations (2a) and (2c) in the Comment by *McDougall and Barker* [2014], which they present as correct expressions for N^2. Thus *McDougall and Barker* [2014] challenges none of the results presented in *King et al.* [2012], which were obtained using scripts from TEOS-10, as listed in Table 1 of our paper.

In *King et al.* [2012, section 2.2], we also made a critical remark about the use of potential density in determining N^2, but we mistakenly grouped the CSIRO-script sw_bfrq.m with the inexact ones. In the following section, we clarify the intent of *King et al.* [2012, section 2.2].

Our misinterpretation of the script sw_bfrq.m arose from its giving in the preamble Eq. 3.7.15 for N^2 from *Gill* [1982, p. 54], which adopts a *constant* reference level for potential temperature. *McDougall and Barker* [2014] clarifies that the method actually used in sw_bfrq.m is a different one, namely the adiabatic leveling method, which correctly uses *local* reference levels, and that the citation in the preamble of sw_bfrq.m should have been to *Millard et al.* [1990]. We thank *McDougall and Barker* for this clarification, and we apologize for our erroneous statement about the correctness of sw_bfrq.m. Further, we regret not contacting the authors of the script prior to the publication of *King et al.* [2012].

2. Potential Density and the Adiabatic Leveling Method

In general, potential density may be defined with respect to any reference pressure, p_r. For given temperature T, (absolute) salinity S and pressure p, we then write the density a water parcel would have if brought adiabatically from its initial level p to reference pressure p_r, i.e., its potential density, as $\rho_r(p, T, S, p_r)$. For given static vertical profiles $T(p)$ and $S(p)$, we introduce the short-hand notation $\rho_r(p, p_r)$, for convenience.

There are now two fundamentally different cases to be distinguished. If the partial derivative is taken with respect to the vertical coordinate (for which hydrostatic pressure p serves as a proxy) and the reference pressure is regarded as a *constant*, say atmospheric value $p_r = p_a$, the resulting expression is *not* proportional to N^2:

$$g^2 \frac{\partial \rho_r}{\partial p}(p, p_a) \neq N^2. \tag{1}$$

The sole aim of *King et al.* [2012, section 2.2] was to reiterate this point, which was made before by, e.g., *Gill* [1982, p. 55].

The other case (not discussed in *King et al.* [2012]) is when p_r is regarded as a *variable*, and is afterward assigned the value $p_r = p$. The resulting expression then is an exact one for local gravitational stability:

$$N^2 = g^2 \left(\frac{\partial \rho_r}{\partial p}(p, p_r) \right) \bigg|_{p_r = p}. \tag{2}$$
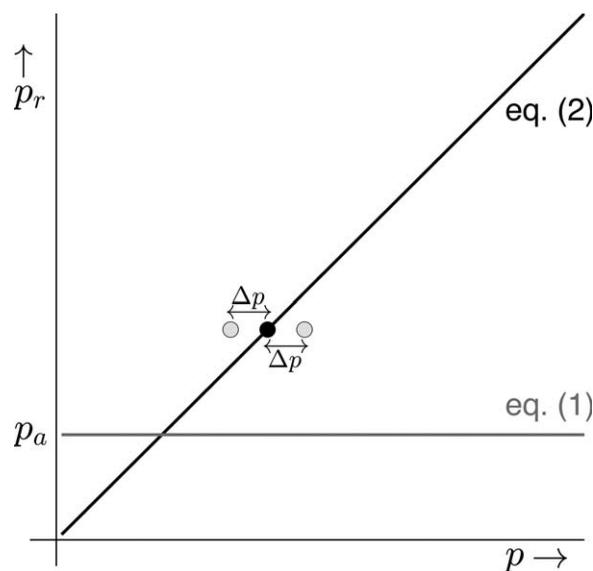
Figure 1. Plane spanned by the independent variables on which the generalized potential density ρ_r depends: initial pressure p and reference pressure p_r. Usually, the latter is taken constant (e.g., at atmospheric pressure), giving the horizontal line at $p_r = p_a$ (in grey), adopted in the evaluation of the left-hand side of (1). If the partial derivative with respect to p is evaluated along the diagonal $p_r = p$ (in black), the result is the identity (2). The circles sketch the discretized way of evaluation proposed by *Millard et al.* [1990].

It is crucial here to take the partial derivative *before* assigning the value to p_r; after all, changing the order would simply result in the vertical gradient of in situ density ρ, since $\rho_r(p,p) = \rho(p)$. Although written differently, equation (2) is equivalent to equation (10) in *McDougall and Barker* [2014].

The difference between how (1) and (2) are evaluated can be illustrated as follows. We regard the independent variables p and p_r in plane coordinates (Figure 1). For potential density with a constant reference level (on the ordinate, p_a, say), one follows the partial derivative with respect to p along a horizontal line. This is the gradient of potential density in the usual sense (i.e., with constant reference level), which is *not* an exact measure of static stability, see (1). If, on the other hand, one follows the partial derivative with respect to p along the *diagonal* $p_r = p$, then one finds the correspondence to (2). This is not a 'gradient' in the common sense of the word, but it does represent an exact expression for N^2. The potential density is now a *local* one, involving a continuum of reference levels.

The "adiabatic leveling method" proposed by *Millard et al.* [1990] casts (2) into a discretized form. At any point on the diagonal (black circle in Figure 1), one evaluates the horizontal partial derivative from a central-difference method using neighboring points at both sides on the horizontal (open circles), so that the right-hand side of (2) is approximated by

$$g^2 \frac{\rho_r(p+\Delta p, p) - \rho_r(p-\Delta p, p)}{2\Delta p}.$$

3. Conclusion

McDougall and Barker [2014] argues for the validity of the adiabatic leveling method for calculating the buoyancy frequency. However, this is not a point of contention; the adiabatic leveling method was not criticized by *King et al.* [2012], it was not even mentioned. As the previous section shows, we fully agree with the validity of that method. Our error was that we failed to realize that the script sw_bfrq.m uses that method. Again, we apologize for our misinterpretation of the script.

In this Reply we have not mentioned section 3 of *McDougall and Barker* [2014], since we fully agree with it. We should however point out that it concerns an older version of the equation of state, EOS-80, one that we did not use in *King et al.* [2012]. We used the modern standard TEOS-10 throughout.

References

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