11.1 Angular Momentum

\[ \vec{L}_{\text{trans}} \equiv \vec{r} \times \vec{p} \]

\[ |\vec{L}_{\text{trans}}| = |\vec{r}| |\vec{p}| \sin \theta \]
11.1 Vector Cross Product

\[ \vec{r} \times \vec{p} = \det \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ r_x & r_y & r_z \\ p_x & p_y & p_z \end{vmatrix} \]

\[ \vec{r} \times \vec{p} = \langle r_y p_z - r_z p_y, -(r_x p_z - r_z p_x), r_x p_y - r_y p_x \rangle \]

\[ \vec{L} = \vec{r} \times \vec{p} = |\vec{r}||\vec{p}| \sin \theta \]
11.1 Direction of Angular Momentum

\[ \vec{L}_{trans} \equiv \vec{r} \times \vec{p} \quad \text{and} \quad |\vec{L}_{trans}| = |\vec{r}| |\vec{p}| \sin \theta \]
Angular Momentum of Halley's Comet

The highly elliptical orbit of Halley's comet is shown in Figure 11.6. When the comet is closest to the Sun, at the location specified by the position vector ("perihelion"), it is $8.77 \times 10^{10}$ m from the Sun, and its speed is $5.46 \times 10^4$ m/s. When the comet is at the location specified by the position vector $\vec{r}$, its speed is $1.32 \times 10^4$ m/s. At that location the distance between the comet and the Sun is $1.19 \times 10^{12}$ m, and the angle $\theta$ is $17.81^\circ$. The mass of the comet is estimated to be $2.2 \times 10^{14}$ kg. Calculate the translational (orbital) angular momentum of the comet, relative to the Sun, at both locations.
Clicker Quiz

Q11.1.m: If an object is traveling at a constant speed in a vertical circle, how does the object's angular momentum change as the object goes from the top of the circle to the bottom of the circle?

1. $|\vec{L}|$ increases
2. $|\vec{L}|$ decreases
3. $|\vec{L}|$ stays the same but the direction of $\vec{L}$ changes
4. The direction and magnitude of $\vec{L}$ remain the same
Clicker Quiz

Q11.1.m: If an object is traveling at a constant speed in a vertical circle, how does the object's angular momentum change as the object goes from the top of the circle to the bottom of the circle?

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3. $|\vec{L}|$ stays the same but the direction of $\vec{L}$ changes

4. The direction and magnitude of $\vec{L}$ remain the same
11.1 Perpendicular Distance
11.1 Perpendicular Distance

\[ r_\perp = |\vec{r}| \sin \theta \]

\[ |\vec{L}| = |\vec{r} \times \vec{p}| = r_\perp |\vec{p}| \]
11.1 Angular Momentum Example

Calculating Angular Momentum
Use three different methods to calculate the angular momentum of the particle in the Figure relative to location B. How does this value compare to the angular momentum of the same particle relative to location A and to location C?
What is the rotational angular momentum of a multi-particle system?

\[ |\vec{L}_{\text{rot}}| = I_{\text{CM}} \omega \]

\[ \vec{\omega} = |\vec{\omega}| \hat{\omega} \]

\[ \vec{L}_{\text{rot}} = I_{\text{CM}} \vec{\omega} \]
11.2 Rotational Kinetic Energy

What is the rotational angular momentum of a multi-particle system?

\[ \vec{L}_{\text{rot}} = I_{\text{CM}} \vec{\omega} \]

\[ K_{\text{rot}} = \frac{1}{2} I_{\text{CM}} \vec{\omega}^2 = \frac{\vec{L}_{\text{rot}}^2}{2 I_{\text{CM}}} \]
11.3 Rotational Kinetic Energy

Center of Mass Motion

\[ \vec{L}_A = \vec{L}_{\text{trans},A} + \vec{L}_{\text{rot}} \]

\[ \vec{L}_{\text{trans},A} = \vec{r}_{\text{CM},A} \times \vec{P}_{\text{tot}} \]

\[ \vec{L}_{\text{rot}} = \vec{r}_{1,CM} \times \vec{p}_1 + \vec{r}_{2,CM} \times \vec{p}_2 + \ldots \]

\[ \vec{P}_{\text{tot}} = \sum_i \vec{p}_i \]

Motion Relative to CoM

\[ K_{\text{tot}} = K_{\text{trans}} + K_{\text{rot}} \]
Clicker Quiz

11.X.8 A rod rotates in the vertical plane around a horizontal axle. A wheel is free to rotate on the rod, as shown in the Figure. A vertical stripe is painted on the wheel. As the rod rotates clockwise, the vertical stripe on the wheel remains vertical. Is the translational angular momentum of the wheel relative to location A zero or nonzero? If nonzero, what is its direction with respect to the screen?
(A) zero (B) nonzero, (in) (C) nonzero, (out)
Clicker Quiz

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Clicker Quiz

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(A) zero  (B) nonzero, (in)  (C) nonzero, (out)

Is the rotational angular momentum of the wheel zero or nonzero? If nonzero, what is its direction?
(A) zero  (B) nonzero, (in)  (C) nonzero, (out)
Clicker Quiz

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(A) zero (B) nonzero, (in) (C) nonzero, (out)

Is the rotational angular momentum of the wheel zero or nonzero? If nonzero, what is its direction?
(A) zero (B) nonzero, (in) (C) nonzero, (out)
11.X.9
Consider a system similar to that in the previous exercise, but with the wheel welded to the rod (not free to turn). Is the translational angular momentum of the wheel relative to location A zero or nonzero? If nonzero, what is its direction?
(A) zero (B) nonzero, (in) (C) nonzero, (out)
Consider a system similar to that in the previous exercise, but with the wheel welded to the rod (not free to turn). Is the translational angular momentum of the wheel relative to location A zero or nonzero? If nonzero, what is its direction?

(A) zero   (B) nonzero, (in) (C) nonzero, (out)

Is the rotational angular momentum of the wheel zero or nonzero? If nonzero, what is its direction?

(A) zero   (B) nonzero, (in) (C) nonzero, (out)
11.X.9
Consider a system similar to that in the previous exercise, but with the wheel welded to the rod (not free to turn). Is the translational angular momentum of the wheel relative to location A zero or nonzero? If nonzero, what is its direction?
(A) zero  (B) nonzero, (in)  (C) nonzero, (out)

Is the rotational angular momentum of the wheel zero or nonzero? If nonzero, what is its direction?
(A) zero  (B) nonzero, (in)  (C) nonzero, (out)
Pinocchio rides a horse on a merry-go-round turning counterclockwise as viewed from above, with his long nose always pointing forward, in the direction of his velocity. Is Pinocchio's translational angular momentum relative to the center of the merry-go-round zero or nonzero? If nonzero, what is its direction?
(A) zero (B) nonzero, (down) (C) nonzero, (up)
Pinocchio rides a horse on a merry-go-round turning counterclockwise as viewed from above, with his long nose always pointing forward, in the direction of his velocity. Is Pinocchio's translational angular momentum relative to the center of the merry-go-round zero or nonzero? If nonzero, what is its direction?

(A) zero   (B) nonzero, (down)  (C) nonzero, (up)

Is his rotational angular momentum zero or nonzero? If nonzero, what is its direction?

(A) zero   (B) nonzero, (down)  (C) nonzero, (up)
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(A) zero  (B) nonzero, (down)  (C) nonzero, (up)

Is his rotational angular momentum zero or nonzero? If nonzero, what is its direction?
(A) zero  (B) nonzero, (down)  (C) nonzero, (up)
Clicker Quiz

11.X.10
Pinocchio rides a horse on a merry-go-round turning counterclockwise as viewed from above, with his long nose always pointing forward, in the direction of his velocity. Is Pinocchio's translational angular momentum relative to the center of the merry-go-round zero or nonzero? If nonzero, what is its direction?
(A) zero (B) nonzero, (down) (C) nonzero, (up)
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(A) zero (B) nonzero, (down) (C) nonzero, (up)

Is his rotational angular momentum zero or nonzero? If nonzero, what is its direction?
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(A) zero (B) nonzero, (down) (C) nonzero, (up)
11.4 The Angular Momentum Principle

\[ \vec{L}_A = \vec{r}_A \times \vec{P} \]
11.4 The Angular Momentum Principle

\[ \vec{L}_A = \vec{r}_A \times \vec{P} \]

\[ \vec{\tau}_A \equiv \vec{r}_A \times \vec{F} \]

\[ \frac{d\vec{L}_A}{dt} = \vec{\tau} \]
Clicker Quiz

A 50 N force acts at a 20 degree angle with respect to the horizontal on a corner of a square that is 3 m long on each side.

What is the torque about the center of the square?

1) $<0, 0, 89.6> \text{N} \cdot \text{m}$
2) $<0, 0, 44.82> \text{N} \cdot \text{m}$
3) $<0, 0, -44.82> \text{N} \cdot \text{m}$
4) $<0, 0, 25.65> \text{N} \cdot \text{m}$
5) $<0, 0, -25.65> \text{N} \cdot \text{m}$
Clicker Quiz

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3) $<0, 0, -44.82>N\cdot m$
4) $<0, 0, 25.65>N\cdot m$
5) $<0, 0, -25.65>N\cdot m$
Clicker Quiz

Consider a horizontal disc of moment of inertia $I_0$ and radius $R$ rotating with angular speed $\omega_0$. A lump of clay of mass $m$ is moving in the plane of the disc, with constant velocity $v_0$. The clay, initially far from the disc, moves toward the disc along a line that passes through the center of the disc. Then the clay hits and sticks to the edge of the disc. Find the angular velocity of the system after the impact.

1. \( \frac{I_0}{I_0 + mR^2} (\omega_0 + v_0/R) \)
2. \( \frac{I_0}{mR^2} \omega_0 \)
3. \( \frac{I_0}{I_0 - mR^2} \omega_0 \)
4. \( \frac{I_0}{I_0 - mR^2} (\omega_0 - v_0/R) \)
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1. $\frac{I_0}{I_0 + mR^2}(\omega_0 + v_0/R)$
2. $\frac{I_0}{mR^2}\omega_0$
3. $\frac{I_0}{I_0 - mR^2}\omega_0$
4. $\frac{I_0}{I_0 - mR^2}(\omega_0 - v_0/R)$
5. $\frac{I_0}{I_0 + mR^2}\omega_0$
11.5 Multiparticle Systems

A Child Jumps onto a Playground Ride
A playground ride consists of a uniform-density disk of mass 300 kg and radius 2 m mounted on a low-friction axle (Figure 11.36). Starting from a distance of 5 m from the disk, a child of mass 40 kg runs at 3 m/s on a line tangential to the disk and jumps onto the outer edge of the disk. If the disk was initially at rest, how fast does it rotate just after the collision?
### 11.6 Three Fundamental Principles

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{d\vec{p}}{dt} = \vec{F}_{\text{net}} )</td>
<td>( \frac{d\vec{L}<em>{\text{tot},A}}{dt} = \vec{\tau}</em>{\text{net},A} )</td>
<td>( \Delta E = W + Q )</td>
</tr>
<tr>
<td>If there are external forces, momentum changes.</td>
<td>If there are external torques, angular momentum changes.</td>
<td>If there are energy inputs, energy changes.</td>
</tr>
<tr>
<td>Location of object does not matter.</td>
<td>Location of object to reference point is important.</td>
<td>Location of object does not matter.</td>
</tr>
</tbody>
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Clicker Quiz

A spool with an inner radius $r$ and outer radius $R$ has a rope wrapped around the smaller radius. If the rope is pulled with tension $T$ in the $+x$ direction, in what direction will the spool move?
A) $+x$ B) $-x$ C) won’t move
Clicker Quiz

A spool with an inner radius r and outer radius R has a rope wrapped around the smaller radius. If the rope is pulled with tension T in the +x direction, in what direction will the spool move?

A) +x  B) -x  C) won’t move

What is the torque relative to the point where the spool contacts the floor?

A) <0, 0, Tr>  B) <0, 0, T(R-r)>  C) <0, 0, -T(R-r)>
D) <0, 0, -Tr>  E) <0, 0, TR>
Clicker Quiz

A spool with an inner radius $r$ and outer radius $R$ has a rope wrapped around the smaller radius. If the rope is pulled with tension $T$ in the $+x$ direction, in what direction will the spool move?

A) $+x$        B) $-x$        C) won’t move

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D) $<0, 0, -Tr>$        E) $<0, 0, TR>$
Clicker Quiz

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A) +x  B) -x  C) won’t move

What is the direction of the torque relative to the point where the spool contacts the floor?

A) +z  B) -z  C) zero
Clicker Quiz

A spool with an inner radius \( r \) and outer radius \( R \) has a rope wrapped around the smaller radius. If the rope is pulled with tension \( T \) in the \(+y\) direction, in what direction will the spool move?

A) \(+x\)  B) \(-x\)  C) won’t move

What is the direction of the torque relative to the point where the spool contacts the floor?

A) \(+z\)  B) \(-z\)  C) zero
11.7 Systems with Zero Torque

$I_i > I_f$

$\vec{\omega}_i < \vec{\omega}_f$

$E_{rot} < E_{rot}$?
Example - A Satellite with Solar Panels
A satellite has four low-mass solar panels sticking out. The satellite can be considered to be approximately a uniform solid sphere. Originally it is traveling to the right with speed $v$ and rotating clockwise with angular speed $\omega_i$. A tiny meteor traveling at high speed $v_1$ rips through one of the solar panels and continues in the same direction but at reduced speed $v_2$. Afterward calculate the $v_x$ and $v_y$ components of the center-of-mass velocity of the satellite and its angular velocity $\omega_f$. 

$$\begin{align*}
M & \text{ mass of the satellite} \\
R & \text{ radius of the satellite} \\
h & \text{ height of the satellite} \\
\theta & \text{ angle of the satellite} \\
m & \text{ mass of the meteor} \\
v_1 & \text{ speed of the meteor before impact} \\
v_2 & \text{ speed of the meteor after impact}
\end{align*}$$
11.7 Systems with Zero Torque

Example - A Satellite with Solar Panels

A satellite has four low-mass solar panels sticking out. The satellite can be considered to be approximately a uniform solid sphere. Originally it is traveling to the right with speed $v$ and rotating clockwise with angular speed $\omega_i$. A tiny meteor traveling at high speed $v_1$ rips through one of the solar panels and continues in the same direction but at reduced speed $v_2$. Afterward calculate the $v_x$ and $v_y$ components of the center-of-mass velocity of the satellite and its angular velocity $\omega_f$. 
11.8 Equilibrium

\[ \frac{d\vec{p}}{dt} = \vec{F}_{\text{net}} = 0 \]
\[ \frac{d\vec{L}_A}{dt} = \tau_{\text{net},A} = 0 \]

Example - Two People on a Seesaw

The person on the left has mass \( M_1 = 90 \text{ kg} \) and sits at a distance \( d_1 = 1.2 \text{ m} \) from the nearly frictionless axle. The person on the right has mass \( M_2 = 40 \text{ kg} \). What is the upward force that the axle must exert? Where must the person on the right sit in order that the seesaw not rotate? The seesaw itself has negligible mass.
Clicker Quiz

A painter who weighs $3W$ is standing on a horizontal board, which is supported by a vertical rope on the left end with tension $T_L$ and by a vertical rope on the right end with tension $T_R$. The board weighs $2W$ and has length $L$. The painter is at distance $L/3$ from the left end.

Find the tension $T_L$.

(A) $3W$   (B) $5W/2$   (C) $3W/2$   (D) $2W$   (E) $W$
A painter who weighs 3W is standing on a horizontal board, which is supported by a vertical rope on the left end with tension $T_L$ and by a vertical rope on the right end with tension $T_R$. The board weighs 2W and has length $L$. The painter is at distance $L/3$ from the left end.

Find the tension $T_L$.

(A) 3W   (B) 5W/2   (C) 3W/2   (D) 2W   (E) W

Answer: Torque about right end is

Net torque = $-LT_L + (2L/3)3W + (L/2)2W + 0xT_R = 0$

So $T_L = 2W + W = 3W$  Answer (A)
Clicker Quiz

A horizontal rod of negligible mass and length L is supported at the right end by a rope that is at angle $\theta$ above the horizontal. A rock of mass M is suspended from the rod at a distance 3/4 of the way from the left end. The left end of the rod is supported by a frictionless hinge.

What is the tension in the rope?

A) $3Mg/(4\tan\theta)$   B) $3Mg/(4\sin\theta)$   C) $Mg/(4\sin\theta)$
D) $Mg/(4\tan\theta)$   E) $Mg/(4\cos\theta)$
A horizontal rod of negligible mass and length L is supported at the right end by a rope that is at angle $\theta$ above the horizontal. A rock of mass $M$ is suspended from the rod at a distance $3/4$ of the way from the left end. The left end of the rod is supported by a frictionless hinge.

What is the tension in the rope?

A) $\frac{3Mg}{4\tan\theta}$  B) $\frac{3Mg}{4\sin\theta}$  C) $\frac{Mg}{4\sin\theta}$

D) $\frac{Mg}{4\tan\theta}$  E) $\frac{Mg}{4\cos\theta}$
11.8 Systems with Nonzero Torques

Example - A Meter Stick on the Ice

Consider a meter stick whose mass is 300 grams and that lies on ice (in the figure we're looking down on the meter stick). You pull at one end of the meter stick, at right angles to the stick, with a force of 6 newtons. Assume that friction with the ice is negligible. What is the rate of change of the center-of-mass speed $v_{CM}$? What is the rate of change of the angular speed $\omega$?

\[ \alpha \equiv \frac{d\omega}{dt} \]
11.9 Prediction Positions in Rotating Systems

\[ \vec{L}_{\text{rot},f} = \vec{L}_{\text{rot},i} + \vec{\tau}_{\text{net}} \Delta t \]

Example - A wheel of radius R and moment of inertia I is mounted on a low-friction axle. A string is wrapped around the edge, and you pull on it with a force F. At a certain time the angular speed is \( \omega_i \).

(a) After a time interval \( \Delta t \), what is the angular speed \( \omega_f \)?

(b) How far did your hand move during this time interval?
11.10 Bohr Model Energy Levels

\[ r = N^2 \frac{\hbar^2}{\left(\frac{1}{4\pi\epsilon_0}\right)e^2m} \]

\[ E = -\frac{\left(\frac{1}{4\pi\epsilon_0}\right)^2 e^4m}{2N^2\hbar^2} \]
11.8 Rotational Dynamics Clicker Quiz

A cord is wrapped around a flywheel of inertia \( I = \frac{1}{2} mR^2 \) and radius \( R \). A mass \( m \) is suspended by the cord. The system is released from rest and the suspended mass falls. What is the acceleration of the suspended mass \( m \)?

1) \( \frac{g}{4} \)
2) \( \frac{2g}{3} \)
3) \( 2g \)
4) \( g \)
5) \( \frac{3g}{2} \)
A cord is wrapped around a flywheel of inertia $I = \frac{1}{2} mR^2$ and radius $R$. A mass $m$ is suspended by the cord. The system is released from rest and the suspended mass falls. What is the acceleration of the suspended mass $m$?

1) $g/4$

2) $2g/3$

3) $2g$

4) $g$

5) $3g/2$
11.8 Statics Example

A uniform ladder of mass 15kg and length 3m leans against a smooth wall at an angle 45° with respect to the ground. A person of mass 50kg climbs up the ladder to a point two-thirds of the way to the top. Assume that there is no friction between the top of the ladder and the wall.

Compute the minimum coefficient of static friction between the ladder and the ground that would prevent the ladder from slipping.
11.8 Rotational Dynamics Example

String is wrapped around an object of mass $M$ and moment of inertia $I$. With your hand you pull the string straight up with some constant force $F$ such that the center of the object does not move up or down, but the object spins faster and faster. This is like a yo-yo; nothing but the vertical string touches the object. When your hand is a height $y_0$ above the floor, the object has an angular speed $\omega$. When your hand has risen to a height $y$ above the floor, what is the angular speed $\omega$ of the object? (eliminate $F$ from your result)

$$\omega_f = \sqrt{\frac{2Mg(y - y_0)}{I}} + \omega_i^2$$
12.2 Statistical Model of Solids

Distributing quantized energy among discrete harmonic oscillators.

\[ E_N = N\hbar\omega_0 + E_0, \quad N = 0, 1, 2, \ldots, \text{where } \omega_0 = \sqrt{\frac{k_s}{m}} \]
THE FUNDAMENTAL ASSUMPTION OF STATISTICAL MECHANICS - for a given macrostate (e.g. certain temperature) over time, an isolated system (oscillator) is **equally likely** to be found in **any** of its possible **microstates** (how quanta of energy are in that oscillator).
12.2 Combinatorics - physics

Arranging 3-quanta into 3 oscillators

\[
\frac{(3 + 3 - 1)!}{3!(3 - 1)!} = 10
\]

Arranging \( q \)-quanta into \( N \) oscillators

\[
\Omega = \frac{(q + N - 1)!}{q!(N - 1)!}
\]
12.3 Einstein Model of a Solid

Each atom has three total oscillators, one in each dimension.
Each atom has three total oscillators, one in each dimension.
12.3 Two Blocks in Contact

Simplest Case - 1 atom each, but four quanta shared between them

<table>
<thead>
<tr>
<th>$q_1$</th>
<th>$q_2$</th>
<th>$\Omega_1$</th>
<th>$\Omega_2$</th>
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<td>4</td>
<td>0</td>
<td>15</td>
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