9.3 Parallel Axis Theorem

Rigid Rotation about a Point Not the Center of Mass

Figure 9.25   An object with known ICM is connected to a low-mass rod and rotates around an axle. The red stripe on the object rotates at the same rate $\omega$ as the rod.

\[ I_{\text{tot}} = I_{\text{CM}} + Mr^2_{\text{CM}} \]
9.3 Parallel Axis Theorem

A Rod Rotating Not Around Its Center
A thin rod with mass 140 grams and 60 cm long rotates at an angular speed of 25 radians/s about an axle that is 20 cm from one end of the rod.
9.3 Parallel Axis Theorem

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A thin rod with mass 140 grams and 60 cm long rotates at an angular speed of 25 radians/s about an axle that is 20 cm from one end of the rod

\[ K_{tot} = \frac{1}{2} M \left( \overrightarrow{r}_{CM}^2 + \frac{1}{12} L^2 \right) \omega^2 = 1.75 \text{ J} \]
9.3 Moment of Inertia $I \equiv \sum \limits_i m_i r_{\perp,i}^2 = \int r_{\perp}^2 dm$

The moment of inertia depends on:
1) Total mass
2) Distribution relative to rotation axis

$I_{\text{cylinder}} = \frac{1}{12} ML^2 + \frac{1}{4} MR^2$

$I_{\text{disk}} = I_{\text{cylinder}} = \frac{1}{2} MR^2$

$I_{\text{sphere}} = \frac{2}{5} MR^2$
9.3 Moment of Inertia \( I \equiv \sum_i m_i r_{\perp,i}^2 = \int r_{\perp}^2 \, dm \)

The moment of inertia depends on:
1) Total mass
2) Distribution relative to rotation axis

\[ I_{\text{cylinder}} = \frac{1}{2} M R^2 \quad I_{\text{sphere}} = \frac{2}{5} M R^2 \]

Why is the cylinder’s moment larger?
9.3 Moment of Inertia

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1) Total mass
2) Distribution relative to rotation axis

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Why is the cylinder’s moment larger?

$$I_{\text{cylinder}} = \frac{1}{2} MR^2 \quad I_{\text{sphere}} = \frac{2}{5} MR^2$$
9.3 Rolling Kinetic Energy

\[ I = \beta M R^2 \quad \omega = \frac{v_{CM}}{R} \]
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\[ I = \beta M R^2 \quad \omega = \frac{v_{CM}}{R} \]

1) All objects have same \( K_{tot} \), regardless of \( \beta \)
2) Objects with larger \( \beta \) have larger \( K_{rot}/K_{tot} \)
3) Objects with larger \( \beta \) have larger \( v_{CM} \)
4) 1 & 2 are true
5) 1 & 3 are true
9.3 Rolling Kinetic Energy

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3) Objects with larger \( \beta \) have larger \( v_{CM} \)
4) 1 & 2 are true
5) 1 & 3 are true
Two spherical objects look the same on the outside and have the same mass, but you know that one of them is hollow. You roll them both down an incline simultaneously. How can you tell which one is hollow?

1) The one that gets to the bottom first is hollow.
2) The one that gets to the bottom last is hollow.
3) You need an additional experiment to differentiate them.
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9.3 Rolling Kinetic Energy

You could also differentiate the spheres using Archimedes Principle (buoyancy) with an apparatus like this one.
1) True
2) False
9.3 Rolling Kinetic Energy

You could also differentiate the spheres using Archimedes Principle (buoyancy) with an apparatus like this one.

1) True
2) False
9.4 Point Particle System

\[
K_{\text{tot}} = K_{\text{trans}} + K_{\text{rel}}
\]

\[
\frac{d\vec{p}_{\text{sys}}}{dt} = \vec{F}_{\text{net}}
\]

\[
\Delta K_{\text{trans}} = \int_{i}^{f} \vec{F}_{\text{net}} \cdot d\vec{r}_{CM}
\]
9.4 Point Particle System

The “point particle” system is different from real system:
They have the SAME:
1) Mass
2) \( \nu_{CM} \)
3) Translational kinetic energy
4) Momentum principle applies to both:
5) Same path

But, NOT THE SAME:
1) \( K_{tot} \)
2) Internal energy
3) Vibration
4) Rotation
5) Thermal energy
9.5 Analyzing PP and Real Systems

\[ \frac{d\vec{p}_{sys}}{dt} = \frac{d}{dt}(M_{\text{total}} \vec{v}_{CM}) = \vec{F}_{net} \]

You pull a puck by a string wrapped around its perimeter with a constant force for some time \( t \). During that time, your hand moves a distance \( \Delta \vec{r}_{tot} \)

What is the rotational kinetic energy of the puck?
9.5 Analyzing PP and Real Systems

A Box Containing a Spring
A thin box in outer space contains a large ball of clay of mass $M$, connected to an initially relaxed spring of stiffness $k_s$. The mass of the box and spring are negligible compared to $M$. The apparatus is initially at rest. Then a force of constant magnitude $F$ is applied to the box. When the box has moved a distance $b$, the clay makes contact with the left side of the box and sticks there, with the spring stretched an amount $s$.

(a) When the clay sticks to the box, how fast is the box moving?
(b) What is the increase of internal energy of the real system?