Comment on "Nonlinear Magneto-Optics of Vacuum: Second-Harmonic Generation"

In a recent Letter, Ding and Kaplan[1] consider the QED effect of second-harmonic generation (SHG) in vacuum in the presence of a strong dc magnetic field. They find that with existing technology, it is now feasible to observe this effect experimentally. In this Comment we show that their calculation is not correct and find that the effect is too small to be observed currently in a laboratory experiment.

The starting point of the calculation of Ref. 1 is the Heisenberg-Euler Lagrangian from which nonlinear terms for the electric displacement **D** and magnetic induction **B** are derived [Eqs. (1) and (2) in Ref. 1]. While this Lagrangian is certainly valid for the case of constant fields, its application to the problem of SHG of collinear photons is not correct. Since we are considering QED effects involving virtual electrons and positrons, the correct starting point for this calculation is the Feynman diagrams with three real collinear photon lines and $2n+1$ external field lines. There are three small parameters in the problem. The first is the fine-structure constant $\alpha = e^2/\hbar c$. The second is $x = H_0/H_c$, where H_0 is the dc magnetic field and $H_c = m^2c^3/e\hbar = 4.4\times10^9$ T is the critical field. The third is $y = \omega/\omega_C$, where ω is the optical frequency and $\omega_C = mc^2/\hbar$ is the Compton frequency. For optical frequencies and magnetic fields currently attainable in the laboratory $\alpha \gg y \gg x$. The derivative expansion of the scattering amplitude is equivalent to an expansion in x and in y. Each term in the expansion can be related to Feynman diagrams with a certain number of real photon lines and external field interactions. The lowest-order diagram is the "box" diagram Fig. 1(a). It has been shown in Refs. 2–5 that this diagram vanishes identically due to symmetries of QED for the case of three collinear photons and one external dc-magnetic-field interaction. Noncollinearity is possible only if energy and momentum are exchanged with the external field which amounts to the addition of real photon lines. We must use more complicated diagrams to calculate the effect. First, note that all diagrams of higher order in α with one dc-magnetic-field interaction and three real photon lines vanish as well since the symmetry arguments of Adler *et al.*[2] do not depend on the order of a diagram. Since $y \gg x$, we must consider adding real photon external lines before adding dc-magnetic-field interactions. We are led to consider diagrams such as Fig. 1(b) corresponding to fourth-harmonic generation $\omega+\omega+\omega+\omega \rightarrow 4\omega$. But a minor refinement of the symmetry arguments of Adler *et al.*[2] shows that this diagram vanishes. Consequently, we

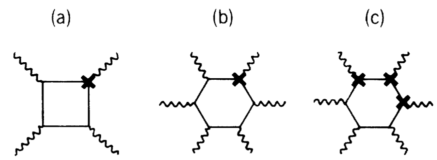

FIG. 1. Relevant Feynman diagrams. Solid lines represent virtual electrons and positrons. Wavy lines represent real photons while wavy lines with crosses represent dc-magnetic-field interactions.

have to consider the addition of dc-magnetic-field interactions. The lowest-order diagram is given in Fig. 1(c), and is the first nonvanishing contribution to SHG in vacuum. A calculation of the effective nonlinearity analogs of Eqs. (1) and (2) of Ding and Kaplan shows that for a given magnetic field H_0 the SHG field is smaller than the values given in Ref. 1 by a factor of order H_0^2/H_c^2. For magnetic fields currently attainable in the laboratory, this factor makes the effect too small to observe experimentally.

We thank Wayne M. Itano and E. Johnson for helpful suggestions. One of us (B.R.) is supported by Natural Sciences and Engineering Research Council of Canada and NSF Grant No. PHY 82-17853.

M. G. Raizen
Time and Frequency Division
National Institute of Standards and Technology
325 S. Broadway
Boulder, Colorado 80303

Baruch Rosenstein
Physics Department
University of British Columbia
Vancouver, British Columbia, Canada V6T 2A6
and
Institute for Theoretical Physics
University of California
Santa Barbara, California 93106

Received 7 February 1990
PACS numbers: 42.50.Wm, 12.20.Ds

[1] Y. J. Ding and A. E. Kaplan, Phys. Rev. Lett. **63**, 2725 (1989).
[2] S. L. Adler, J. N. Bahcall, C. G. Callen, and M. N. Rosenbluth, Phys. Rev. Lett. **25**, 1061 (1970).
[3] Z. Bialynicka-Birula and I. Bialynicki-Birula, Phys. Rev. D **2**, 2341 (1970).
[4] V. B. Berestetskii, E. M. Lifshitz, and L. P. Pitaevskii, *Quantum Electrodynamics* (Pergamon, New York, 1982).
[5] R. J. Stoneham, J. Phys. A **12**, 2187 (1979).