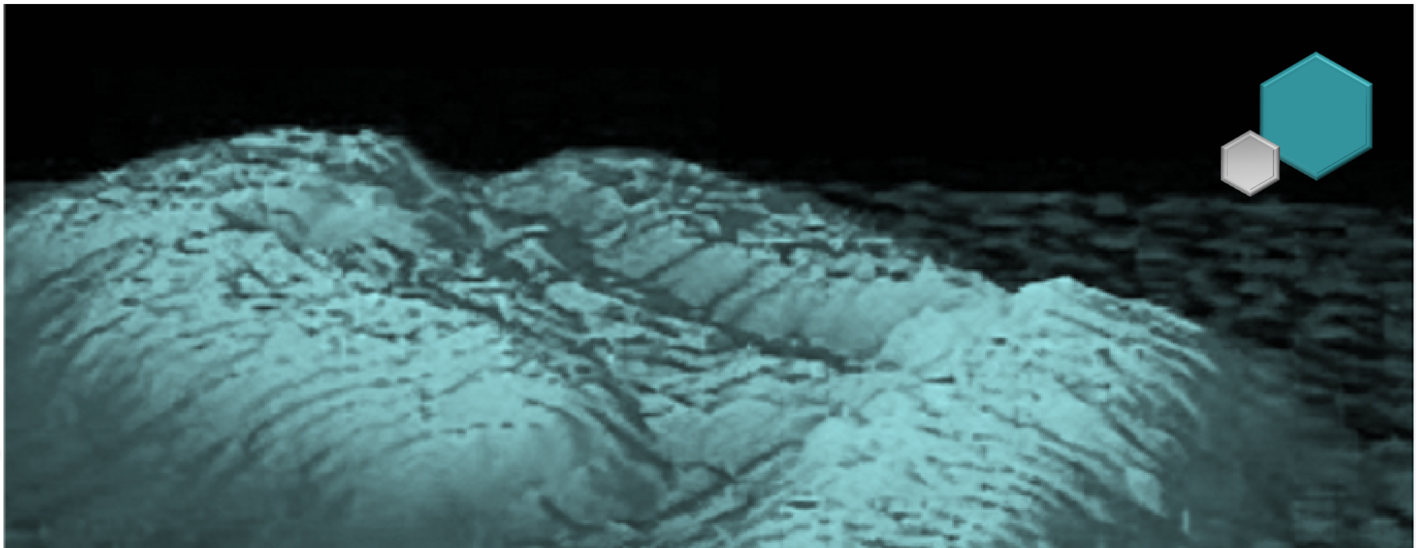
PHYSICS: Atomic Force Microscopes

How does an Atomic Force Microscope work?



PARTS 1-4:

- **1: Understanding How Forces Impact Cantilevers & Springs**
- **2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers**
- **3: Instrument Design: Understanding How AFMs Use Optics & Reflected Light to Determine Sample Properties**
- **4: Instrument Design: Statistical Analysis with Instrumentation**

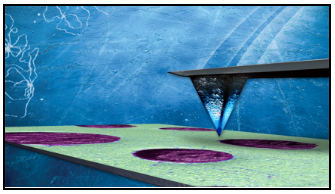
Image Credit: Atomic Force Microscope Image, Eschericia coli--. C.J. Kazilek, Arizona State University. (2001). https://www.nsf.gov/news/mmg/mmg_disp.jsp?med_id=51835&from=search_list

Files Included for this Activity	5E Lesson Plans*	Activity Lab Sheets
PART 1: Understanding How Forces Impact Cantilevers & Springs		√
PART 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers		√
PART 3: Instrument Design: Understanding How AFMs Use Optics & Reflected Light to Determine Sample Properties		√
PART 4: Instrumentation Design: Statistical Analysis with Instrumentation		√

*5E lesson plans are available under separate cover.

This module was developed by Professors Vernita Gordon and Alexandra Eusebi, with assistance from research & UTeach intern Khusbu R. Dalal, at the University of Texas, Austin, with funding provided by the National Science Foundation, Division of Civil, Mechanical, and Manufacturing Innovation, award numbers 1727544 and 2150878, to Vernita Gordon.

If you use any part of this module, please send an email describing your experience to Professor Gordon, gordon@chaos.utexas.edu. Please include the approximate number of students taught. Documenting this module's use and effectiveness will help us obtain more funding for outreach and education in the future.



[Image source](#)

Atomic Force Microscopes (AFMs)

PART 1: Understanding How Forces Impact Cantilevers & Springs

Introduction

The **atomic-force microscope (AFM)** is a powerful tool that can image almost any surface, including polymers, ceramics, composites, glass, and biological samples. AFMs use a **cantilever** as part of the instrumentation design. A cantilever is a long projecting beam fixed at one end and can be conceptualized as a spring. If you have one end fixed and you displace the other free end up or down, the resulting force from the cantilever will look like a spring force. Using a sharp 10 to 20 nm diameter tip attached to a cantilever, the AFM takes advantage of the atomic forces between the probe and the sample's surface. As the tip of the AFM probe moves in response to tip-sample interactions, the cantilever beam experiences vertical **deflections.** Deflection is the displacement, or a shift, in the object's location from the original position due to the application of force. In an AFM, an optical system that includes focusing a laser beam with a photodiode measures this deflection. In the process, the AFM can measure and localize many forces, including adhesion, magnetic forces, and mechanical properties, to create a 3-D surface profile of the sample.

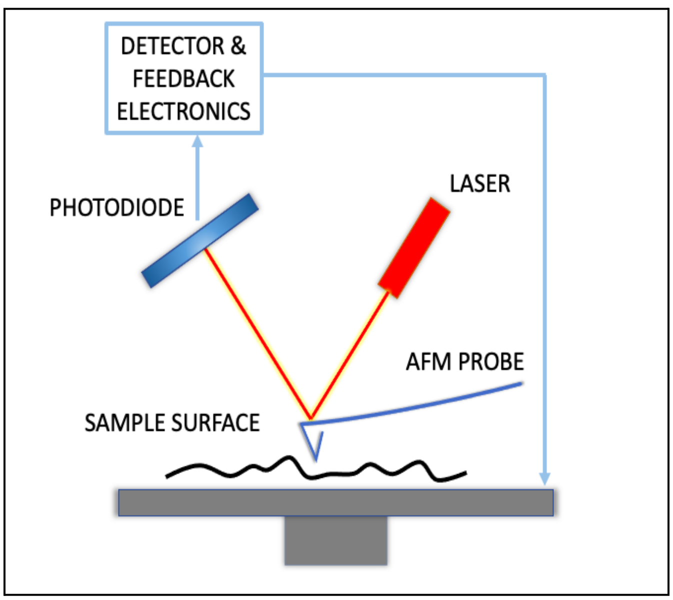


Figure 1: *Atomic force microscope (AFM) schematic.*

This activity will provide insight into how an AFM works by qualitatively investigating the impact of force applied to a cantilever. The same deflection-force relationship that occurs in cantilevers is also exhibited in springs.

In this lab, you will learn how to:

- identify the forces acting upon a cantilever.
- demonstrate and describe qualitatively how force causes deflection of a cantilever.
- describe qualitatively how a material's adhesive properties impact the force required to detach it from a surface.

Activity: Understanding How Force Impacts Cantilever & Springs

Safety Precautions:

- Be mindful of your surroundings when using a meter stick to avoid collisions.
- Avoid putting excessive force on the meter stick, causing it to break. If the meter stick breaks, be careful of the sharp end and dispose of it immediately.

For this activity, you will need the following:

- Wooden meter stick
- Roll of masking or painter's tape (2″ width; if 2″ tape is not available, double 1″ masking tape)
- Roll of packing tape (2″ width)
- Roll of duct tape (2″ width)
- Scissors
- Writing utensil

For this activity you will work *in teams of three or four*.

Activity Introduction

During this activity, you will be exerting force on a cantilever attached to a table with tape. Your cantilever will be a meter stick. Your team will conduct the experiment three times, using different types of tape for each attempt. Throughout the investigation, consider each trial independently, but also think about how the new trials compare to the previous ones.

Pre-Lab Questions

- First, carefully read the description and questions in the Procedure and Observation sections.
- Next, answer the pre-lab questions before starting the lab.

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1. What do you notice or wonder about the setup?

2. What forces are acting upon the system shown in Figure 2 (before attempting to lift the meter stick)? What forces are acting upon the system when you lift the meter stick?

3. What do you anticipate will happen to the meter stick when you apply force? What do you think will happen to the setup as a whole?

4. Do you anticipate all three tapes will hold the same? Do you think they will behave the same? Why or why not?

5. What are the intended take-aways from this activity?

Procedure

Step 1: Start with the masking tape. Cut three pieces of masking tape, each approximately 10 cm in length.

Step 2: With 70 cm of the meter stick extending past the edge of the table, use the three tape pieces to attach the meter stick to the table. Place the tape pieces from 0-5 cm, 13-18 cm, and 25-30 cm of the meter stick.

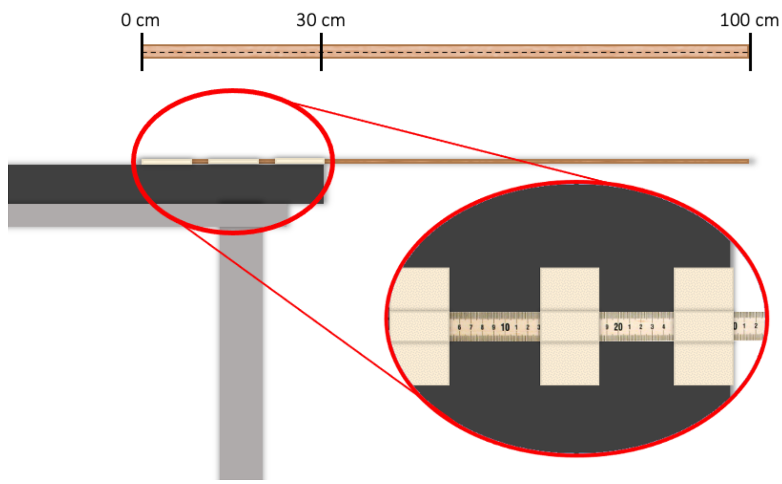


Figure 2: *Experimental setup with 2" masking tape.*

Step 3: With your hand centered around the 90 cm mark, gently lift the meter stick with constant force until the meter stick detaches from the table.

Step 4: Reaffix the tape and allow the next team member to try the activity. Repeat until each team member has participated in the activity. Record your observations in Table 1.

Step 5: Remove the masking tape and repeat the activity using the packing tape. Again, each team member should attempt the activity while the meter stick is affixed with packing tape. Record your individual observations in Table 1 after each team member has completed their turn.

Step 6: Remove the packing tape and repeat the activity using the duct tape. Again, each team member should attempt the activity while the meter stick is affixed with duct tape. Record your individual observations in Table 1 after each team member has completed their turn.

Observations

1. In Table 1 below, describe how you lifted the meter stick. Did you hold the meter stick from above and pull upward, or did you hold it from underneath and bring your forearms up (like a bicep curl)? Does how you hold and lift the meter stick impact the outcome?

Table 1: Qualitative Observations of Applied Force on a Cantilever

OBSERVATIONS	Masking Tape (or Painter's Tape)	Packing Tape	Duct Tape
	What does the process of lifting the meter stick feel like in your muscles?		

2. Describe changes to the meter stick and tape throughout the experiment.

3. Sketch what the system looked like right before the tape was pulled off the table.

4. Was it equally difficult to remove the meter stick when it was taped down with the different types of tape? Explain why or why not.

5. How does this activity better explain deflection associated with an applied force on a cantilever?

6. Based on your observations, describe qualitatively the force exerted by the different types of tape on the system. What does it imply to say they all exerted the same force on the system? Is this accurate?

Extension Activities

In the activity above, many of the variables have been specified for you, including the length of the cantilever, the types, width, and location of the tape, and the position you exert force. Consider what would happen if you changed one or more of those parameters.

- Purpose three possible changes to the system to test. How do you think this would impact the system?
- Hypothesize what will happen when force is exerted on the new system.
- Test your hypothesis and record your observations in Table 1.2.

EXTENSION	

	Suggestions for modifications
	
	• *Modify the adhesive*
	  ○ *Test different types of adhesive (ex., Command™ Strips, poster mounting putty, washi tape, Velcro®, etc.)*
	  ○ *Increase or decrease the amount of tape/adhesive*
	  ○ *Change the location of the tape/adhesive*
	• *Modify the cantilever*
	  ○ *Increase or decrease the length of the cantilever*
	• *Modify the location of the force exerted on the system*

Table 1.2: Extension of Qualitative Observations of Applied Force on a Cantilever

	Trial 1	**Trial 2**	**Trial 3**
Proposed Change			
Hypothesis			
Observations			
Did your observations support your hypothesis? Why or why not?			

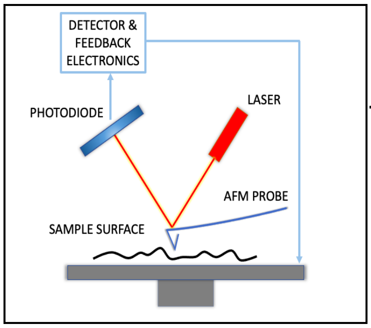
Assessment

1. How is deflection related to force? Explain your thinking.

2. Describe the forces that act upon a cantilever.

3. In what way is the resulting change in the system of force exerted on a cantilever similar to changes that would result in a spring when force is exerted upon it?

4. How do the adhesive properties of a material impact the force required to detach it from a surface?



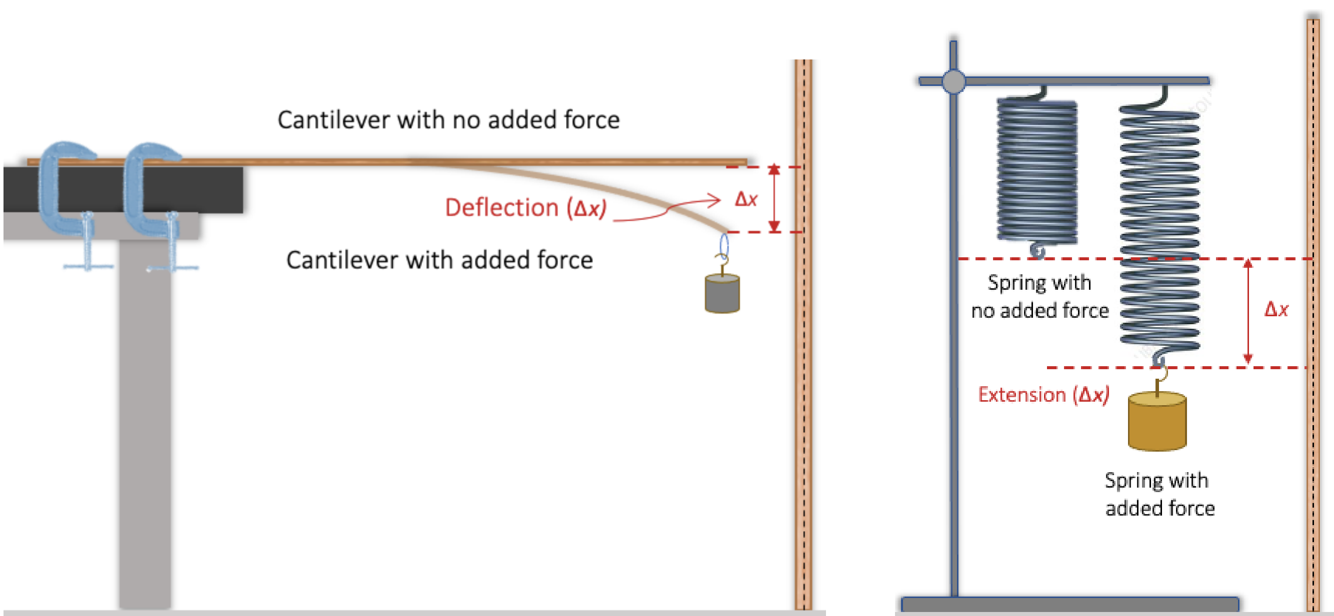
Atomic Force Microscopes (AFMs)

PART 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

Introduction

Atomic force microscopes (AFM) use a cantilever to measure and localize many forces, including adhesion, magnetic forces, and mechanical properties, to create a 3D surface profile of the sample. Much like a spring, the magnitude of the cantilever's deflection, or displacement, depends on the beam's material and the force applied to it.

With cantilevers, **Hooke's law** mathematically relates the **deflection** (Δx) of a cantilever when weight is added to the **force** (F) exerted by the cantilever to keep the system in equilibrium. Similarly, with springs, Hooke's law relates the force applied to an unstretched spring and the amount it stretches. Although some materials may seem similar, each system exhibits Hooke's law differently due to an inherent characteristic called the **spring constant** (k). Figures 1a & 1b show Hooke's law for cantilever and spring in action, respectively.



Figures 1a & 1b: *Displacement of a cantilever and spring due to application of mass on the system.*

We can validate Hooke's law by adding mass to a cantilever system and measuring the resulting deflection. This activity will provide insight into how an AFM works by quantitatively investigating the impact of force applied to a cantilever and deriving Hooke's law for a given system.

In this lab, you will learn how to:

- Identify and illustrate the forces acting upon a mass suspended from a cantilever by drawing and labeling a free body diagram for the system in equilibrium.
- Graphically represent the relationship between deflection and mass added to the free end of a cantilever.
- Mathematically model the behavior of a cantilever system with an equation and graph, developing Hooke's law and recognizing the spring constant.

Activity: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

Safety Precautions:

- Be mindful of your surroundings when using a meter stick to avoid collisions.
- Avoid putting excessive force on the meter stick and breaking it. If the meter stick breaks, be careful of the sharp end and dispose of it immediately.
- Be cautious when putting weights on the meter stick. Heavy weights can cause injury when dropped. Avoid standing or putting your hands in locations in which weights could fall.
- Be cautious when using the clamps so as not to pinch fingers.

For this activity, you will need the following:

- 2 wooden meter sticks
- 2 C-clamps
- weights (ranging in mass: 0 g, 50g, 100 g, 150g, 200 g, 250g, 300 g)
- weight holder or string
- tape
- writing utensil

For this activity you will work *in teams of three or four*.

Activity Introduction

During this activity, you will apply force on the free end of a cantilever attached to a table with a fixed clamp. Your cantilever will be a meter stick. Your team will conduct the experiment five times, using different masses for each attempt. Throughout the investigation, consider each trial independently and how the new trials compare to the previous ones.

Pre-Lab Questions

- First, carefully read the description and questions in the Procedure and the Data, Observation, & Analysis sections.
- Next, answer the pre-lab questions before starting the lab.

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1. What do you notice or wonder about the initial setup?

2. What forces are acting upon the cantilever shown in Figure 2 (before adding mass to the meter stick)? What forces are acting upon the mass when added to the free end of the meter stick?

3. What do you anticipate will happen to the meter stick when you apply force to the free end of the meter stick? Explain your thinking.

4. In this lab, you will derive Hooke's law – the mathematical relationship between force exerted upon a cantilever or spring and the resulting displacement. What mathematical relationship do you hypothesize would best describe this relationship (ex., proportional, inversely related, quadratic)? Explain your thinking.

5. What do you hypothesize will happen to the cantilever when we remove the mass from the system? Explain your thinking.

6. What are the intended takeaways from this activity?

Procedure

Step 1: Using the C-clamps, attach the meter stick horizontally to a table or desk surface with 70 cm of the meter stick hanging over the edge of the surface, as shown in Figure 2.

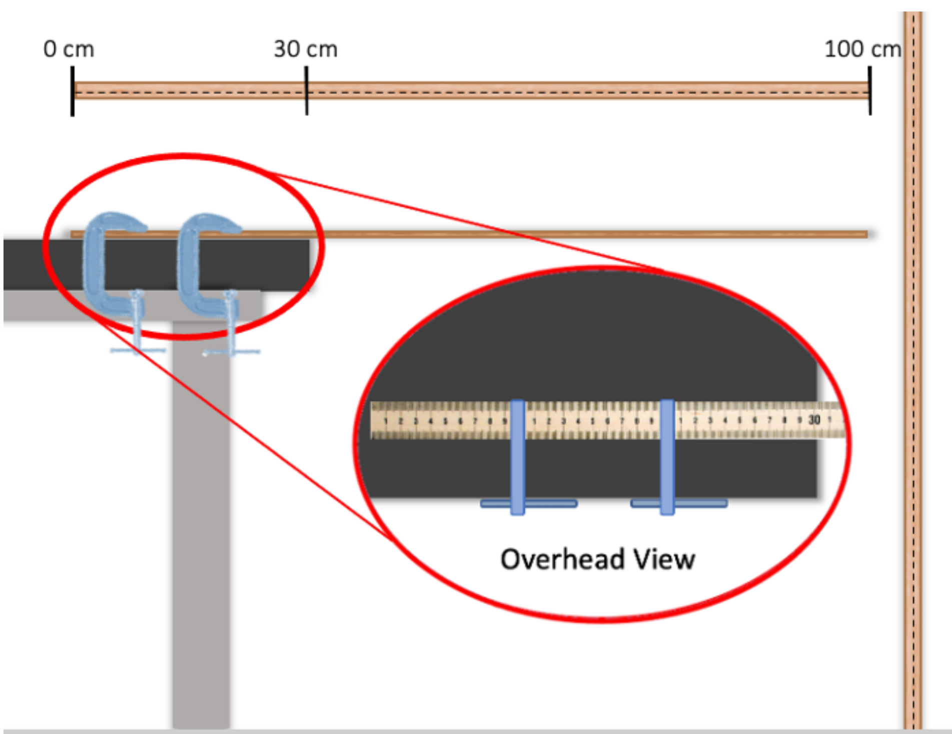


Figure 2: *Experimental setup*

Step 2: Place the second meter stick near the cantilever (perpendicular to the floor with the 0 cm at the floor.) Measure the height of the free end of the meter stick in centimeters to the closest millimeter. Record the initial height value in Table 1 as x_i (corresponding to 0 g of added mass).

Step 3: Tie a length of string to the meter stick's free end. You can secure the string by wrapping it several times around the end of the meter stick and tapping it.

Step 4: Attach the 100 g weight to the string and measure the height of the free end of the meter stick, measuring upward from the floor. Record the mass of the added weight (in grams) and corresponding final height x_f of the end of the meter stick in Table 1.

Step 5: Repeat Step 4 using 100 g, 150 g, 200 g, 250 g, and 300 g.

Data, Observations, & Analysis

7. Draw and label two free body diagrams of the mass: one while the mass is not attached to the cantilever and the second with the mass suspended from the cantilever. Be sure to include the force of weight.

8. As you work through the lab, record the mass of the attached weights and the resulting height of the cantilever in Table 1. Calculate the vertical deflection by subtracting the new height of the cantilever's end after each trial from the initial height ($\Delta x = x_i - x_f$.)

Table 1: Data Collection

Mass of Attached Weight, *m* (g)	Vertical Height, x_f (cm)	Vertical Deflection, $\Delta x = x_i - x_f$ (cm)*
0	x_i =	0
100		
200		
300		
400		
500		

For m= 0 g, x_i=x_f ,therefore, Δx = 0 cm.

9. Calculate the force (F) exerted on the cantilever by the added masses for each trial. Record the values in Table 2. Recall that force is equal to mass times acceleration (F = m·a), where a is the acceleration due to gravity (a = 9.81 m/s^2). Copy the vertical deflection from Table 1 to Table 2, **converting centimeters to meters**. Note that the mass units have been converted to kg to report force in terms of Newtons (N = kg·m/s^2).

Table 2: Data Processing

Mass of Attached Weight, m (kg)	Force Exerted by Cantilever on the weight, F = m·a (N)	Vertical Deflection, $\Delta x = x_i - x_f$ (m)*
0.000	0	0
0.100		
0.200		
0.300		
0.400		
0.500		

**Report Δx in meters.*

10. On the provided graph (Figure 3), label the x-axis as "Vertical Deflection (Δx)" and the y-axis as "Force (F)." Plot the deflection as a function of added mass. Refer to the deflection distance using the symbol Δx.

Modeling Hooke's Law

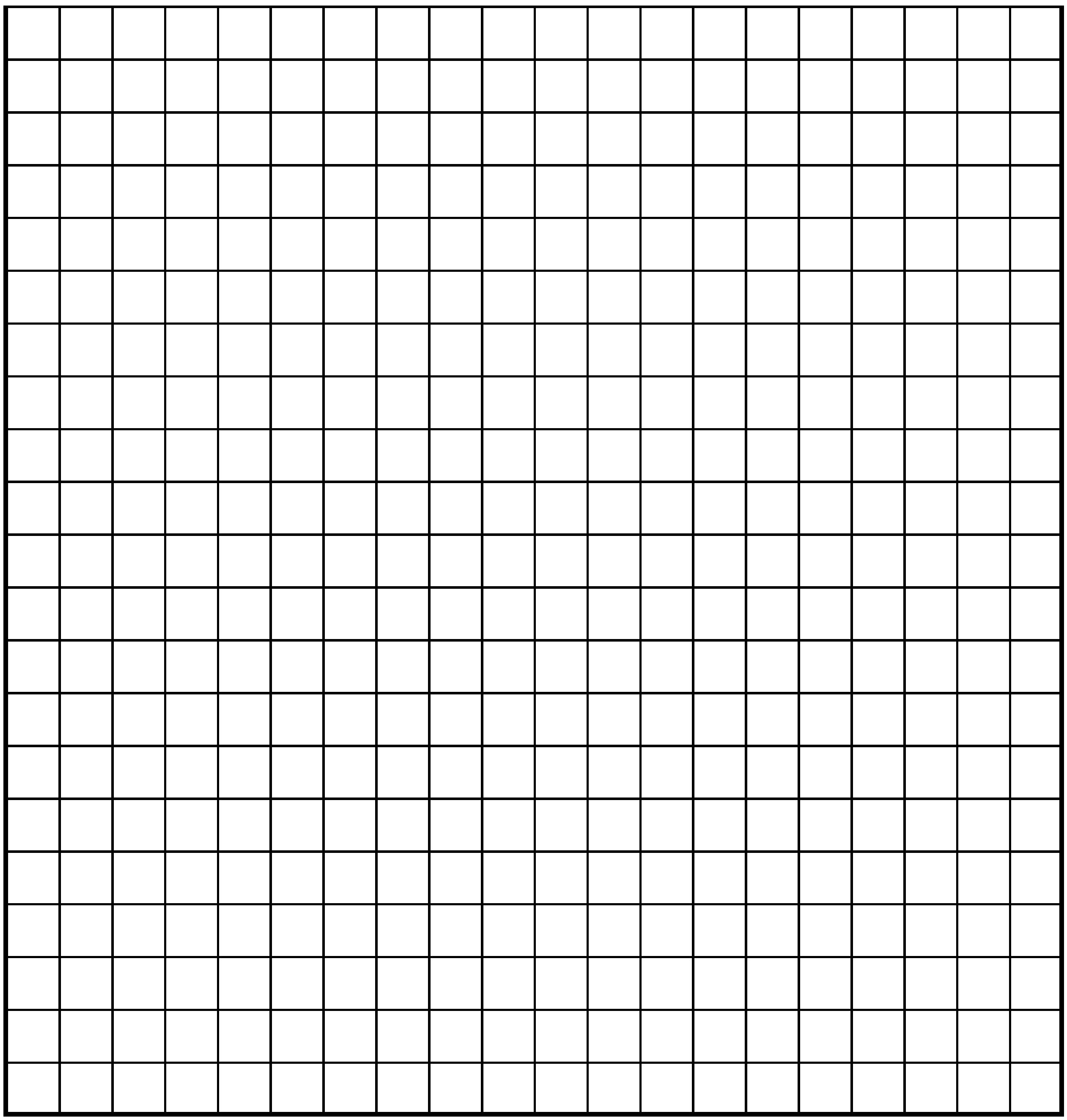


Figure 3. *Force versus Vertical Deflection*

11. Draw a line of best fit through the data points. What type of relationship best describes the graph you drew?

12. Determine the slope of the line of best fit from your graph. Be sure to include units. Give this slope the symbol k. How does your k value compare to those found by groups around you? What does the slope of the line of best fit represent? Use the units of the slope in the description of its meaning. Relate the meaning of the slope to this particular lab and meter stick. Use your specific slope as an example.

13. Determine an equation that best represents the function shown on your graph. Be sure to use the symbols F and Δx. What is your y-intercept, and what does it mean? Should your equation have a y-intercept of 0 Newtons? Explain why or why not.

Extension Activities

1. Consider what would happen if you used a different cantilever. How would the results change if the meter stick was thinner or thicker? What if you used a cantilever with a completely different shape, such as a dowel rod? How would you expect the results for a metal meter stick to differ?

   Repeat the experiment with a different cantilever and hypothesize how the results would compare to the results for the original experiment. After completing the experiment, comment on if your hypothesis was correct or incorrect. Explain your answer.

Table 3: Extension – Data Collection

Mass of Attached Weight, *m* (g)	Vertical Height, x_f (cm)	Vertical Deflection, $\Delta x = x_i - x_f$ (cm)*
0	x_i =	0
100		
200		
300		
400		
500		

For m= 0 g, x_i=x_f ,therefore, Δx = 0 cm.

Table 2: Extension – Data Processing

Mass of Attached Weight, m (kg)	Force Exerted by Cantilever on the weight, F = m·a (N)	Vertical Deflection, $\Delta x = x_i - x_f$ (m)*
0.000	0	0
0.100		
0.200		
0.300		
0.400		
0.500		

Report Δx in meters.

2. On the provided graph (Figure 4), label the x-axis as "Vertical Deflection (Δx)" and the y-axis as "Force (F)." Plot the deflection as a function of added mass. Refer to the deflection distance using the symbol Δx.

Extension - Modeling Hooke's Law

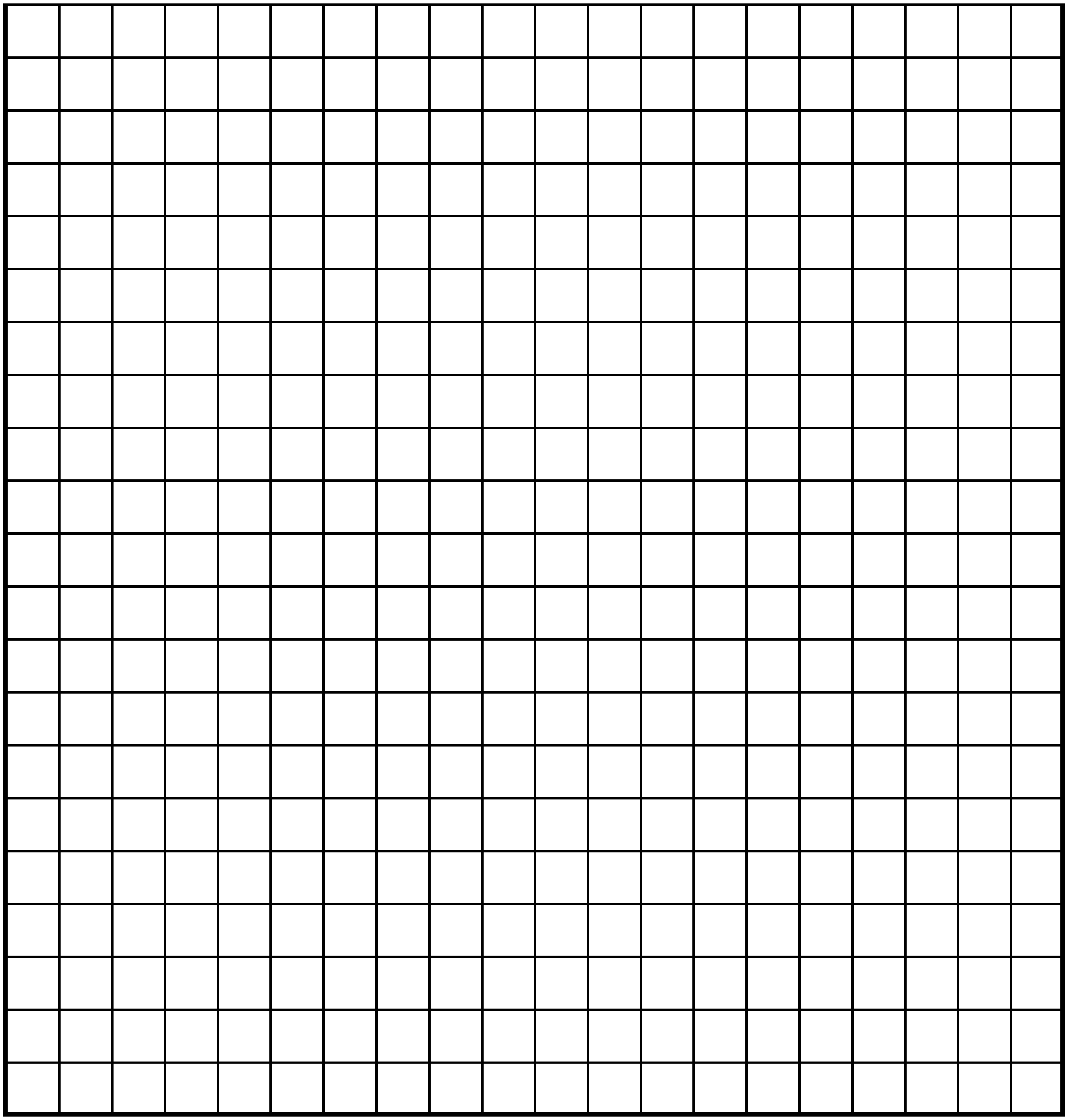


Figure 4. *Extension - Force versus Vertical Deflection*

3. In the Data, Observation, & Analysis section, we asked you to draw a free-body diagram of the mass cantilever system. As an extension, draw a free body diagram of the cantilever in the system with mass added. Include the following: the force of the clamp, the force of gravity (for the hanging and clamped portions), applied force, and normal force.

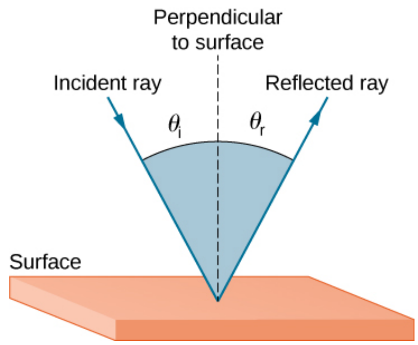
Assessment

1. Using your equation from question 13 in the Data, Observations & Analysis section, create a generalized formula that relates force, displacement, and the spring constant. Explain your reasoning.

2. What variables must be valid for a system to exhibit Hooke's law?

3. How is the resulting change in the system of force exerted on a cantilever similar to changes that would result from a force exerted on a spring?

4. What would Hooke's law look like if we used a less flexible cantilever? What about a more flexible cantilever? Explain your thinking. Can you think of a general rule for relating a material's physical characteristic to the spring constant?



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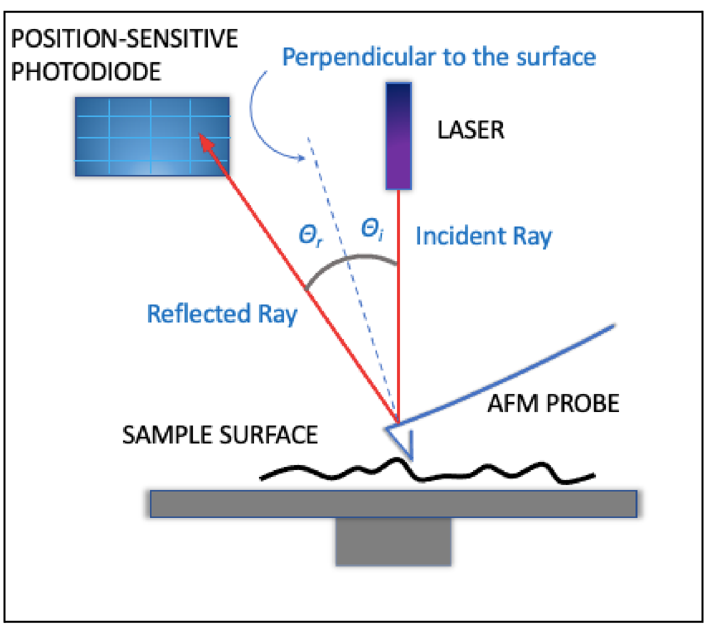
Atomic Force Microscopes (AFMs)

PART 3: Instrument Design: Understanding How AFMs Use Optics & Reflected Light to Determine Sample Properties

Introduction

Atomic force microscopes (AFMs) allow us to study samples at the nanoscale to measure and localize many forces, including adhesion, magnetic forces, and mechanical properties, to create a 3D surface profile of the sample. To do this, the AFM uses a cantilever system with a very sharp tip to traverse the surface of a material. Laser light is directed at a small mirror at the end of the cantilever. The **incident ray** then reflects onto a **position-sensitive photodiode (PSPD)**. As the cantilever deflects from forces exerted on the tip, slight changes occur in the **reflected ray**. The PSPD registers any changes in the position the reflected ray hits the photodiode. From this information and the known unchanging path of the incident ray from the laser, the **angle of incidence Θ_i** and **angle of reflection Θ_r.** can be calculated, and changes in these values are tracked. As the AFM passes over a raised surface, the PSPD records the change in the position of the reflected ray, and how that position is changing. From this information, we can calculate the change in the angle of incidence and the change in the cantilever deflection.



Figures 1: *Diagram of the optics system of an AFM.*

We can better understand how an AFM uses incident light to map properties of a sample's surface by adding mass to a cantilever system, shining a laser onto a mirror attached beneath the cantilever, and measuring the position of the reflected ray. As the cantilever tracks the surface of the sample, the cantilever deflects. This results in changes to the angle of incidence for the laser light shining onto the mirror – and, therefore, the angle of reflection. In turn, the position of where the reflected light hits the photodiode will change as the cantilever deflects under different loads.

In this lab, you will learn how to:

- identify and illustrate the path and angle of the reflected and incident rays.
- graphically represent the relationship between the change in the angle of the incident light and the mass added to the free end of a cantilever.
- graphically represent the relationship between the change in the angle of the incident light and deflection when mass is added to the free end of a cantilever.

Activity: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

Safety Precautions:

- Mirrors are generally made of glass, so handle them carefully.
- Do not look directly into the laser; the light can damage your eye.
- Do not aim the laser at anyone's face.
- Keep the laser turned off when not taking measurements.
- Be mindful of your surroundings when using a meter stick to avoid collisions.
- Avoid putting excessive force on the meter stick and breaking it. If the meter stick breaks, be careful of the sharp end and dispose of it immediately.
- Be cautious when putting masses on the meter stick. Heavy masses can cause injury when dropped. Avoid standing or putting your hands where the masses could fall.
- Be cautious when using the clamps so as not to pinch fingers.

For this activity, you will need the following:

- 2 wooden meter sticks
- 1 metric ruler
- 1" x 1" mirror
- 2 C-clamps
- laser pointer (red or green)
- ring-stand with 2 clamps (to hold the laser pointer and one of the meter sticks).
- weights (ranging in mass: 0 g, 50g, 100 g, 150g, 200 g, 250g, 300 g)
- weight holder or string
- two pieces of printer paper
- tape
- colored pencils & highlighters
- writing utensil

For this activity, you will work *in teams of three or four*.

Activity Introduction

During this activity, you will use mass to apply force on the free end of a cantilever attached to a table with a fixed clamp. Your cantilever will be a meter stick with a mirror attached to the underside of the free end. You will set up the laser pointer to shine vertically upward to the mirror. Your team will modify the system five times, using different masses for each attempt. Throughout the investigation, consider each trial independently and how the new trials compare to the previous ones.

Pre-Lab Questions

- First, carefully read the description and questions in the Procedure and the Data, Observation, & Analysis sections.
- Next, answer the pre-lab questions before starting the lab.

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1. What do you notice or wonder about the initial setup?

2. Consider the setup before adding mass. What forces are acting on the cantilever? What forces are acting on the mass? Sketch and label the system. Use a different colored pencil or marker to indicate the direction of the various forces acting upon the system.

3. On the same diagram, draw what you anticipate will happen to the slope of the mirror when you apply force to the free end of the meter stick? Is there a limit to how you think the mirror will change? Explain your thinking.

4. Considering the lab setup and your previous experience with cantilever systems.
    a. How much deflection do you anticipate will occur with 100 g of added mass?

    b. What slope would you anticipate the mirror to be with this added weight relative to the horizontal axis? Sketch your prediction below.

    c. In the sketch above, use different colored pencils to draw the laser hitting the mirror. Use an additional color to sketch a dashed line indicating the direction of the line perpendicular to the mirror at the point the incident ray hits it.

    d. In the sketch above, continue the path of the incident ray to show your prediction for the path of the reflected ray. Where do you anticipate it will hit the ground? In your image, label the angle of incidence θ_i and the angle of the reflected ray θ_r .

5. Label your sketch with the vertical height component y_f of the cantilever and the horizontal distance of the reflected beam x_f . Letting $\alpha = \theta_r + \theta_i = 2\theta_i$, use trig functions to write an equation representing the relationship between x_f , y_f , and α. Rewrite the equation in terms of θ_i.

6. Do you predict any correlation between the variables x_f, y_f, θ_i, and the added mass? What relationship do you hypothesize exists between the variables and the added mass? Explain your thinking.

7. What are the intended takeaways from this activity?

Procedure

Step 1: Experimental Setup
Cantilever: Using the C-clamps, attach the meter stick horizontally to a table or desk surface with 70 cm of the meter stick hanging over the edge of the surface, as shown in Figure 2. Attach the mirror to the free end of the meter stick with 5 cm of free space at the end. Tie a length of string to the meter stick's free end. You can secure the string by wrapping it several times around the end of the meter stick and tapping it.

Laser: Attach the laser pointer to the ring stand with a clamp with the ray points vertically upward. *Use caution not to shine the laser at anyone or to look at it.* Place the ring stand directly under the mirror. Tape two pieces of paper together on the floor with the edge of one piece under the ring stand. You will record the position of the reflected beam on the paper as the lab progresses.

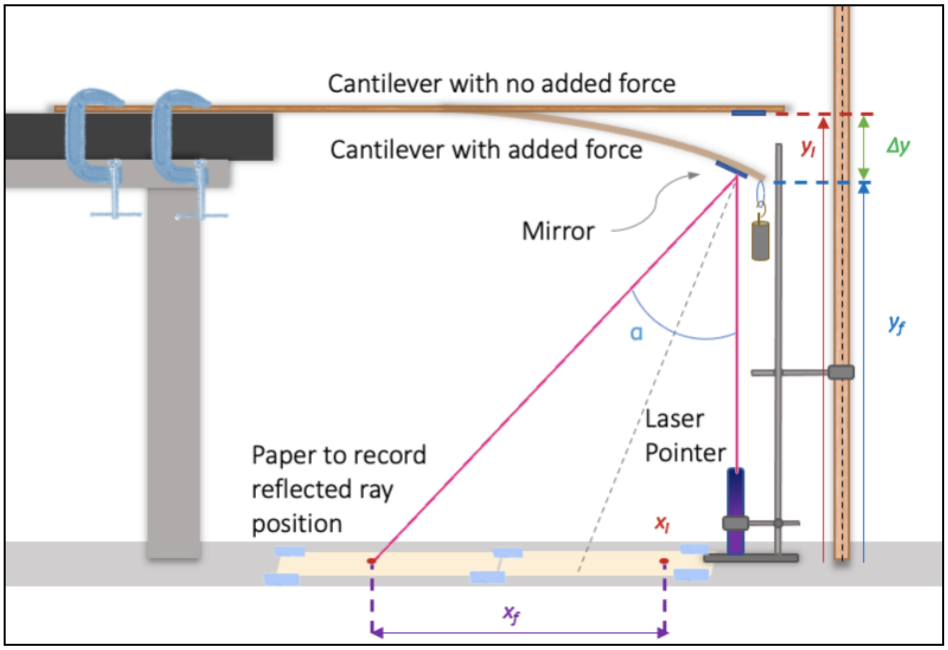


Figure 2: *Experimental setup*

Step 2: Take initial measurements
Cantilever height: Attach the second meter stick to the ring stand (perpendicular to the floor with the 0 cm at the floor.) Place the ring stand near the free end of the cantilever. Measure the height of the free end of the meter stick in centimeters to the closest millimeter. Record the initial height value in Table 1 as y_i (corresponding to 0 g of added mass).

Reflected light: With the laser pointer turned on, carefully mark where the reflected beam hits the paper. Label this point x_i. We will measure the distance the reflected beam hits the floor from this position as mass is added to the cantilever.

Step 3: Exert force on the cantilever & record new height
Attach the 50 g mass to the string and measure the height of the free end of the meter stick, measuring upward from the floor. Record the mass (in grams) and corresponding final height y_f of the end of the meter stick in Table 1.

Step 4: Record the location of reflected light
Mark the location of the reflected ray on the paper. Measure the horizontal distance of the new reflected ray from the initial position x_i to the nearest millimeter and record it in centimeters in Table 1.

Step 4: Repeat Steps 3 and 4 using 100g, 150g, 200g, 250g, and 300g.

Data, Observations, & Analysis

1. Sketch the cantilever set up with 100g of mass impacting the system. Discuss how your prediction from the pre-lab questions resembles the current setup with 100g of added mass. Are they different? How so? Explain your thinking.

2. Sketch the cantilever set up with 300g of mass impacting the system. Qualitatively compare the slope of the mirror in the two scenarios.

3. As you work through the lab, for each trial:

    a. Record the mass *m* and the resulting height of the cantilever x_i in Table 1. Calculate the deflection Δy ($\Delta y = y_i - y_f$).

    b. Record the horizontal distance of the reflected ray relative to the initial position x_i.

    c. Calculate the ratio of the vertical position to the horizontal position using x_i / y_i.

4. Using your trig equation from pre-lab question 5, calculate θ_i for each added mass. Record these values in Table 1 in degrees.

Table 1: Data Collection & Analysis

Mass of Attached Weight, *m* (g)	**Vertical Height, y_f (cm)**	**Vertical Deflection, Δ*y* (cm)**	**Horizontal Position of the Reflected Ray, x_f (cm)***	**Ratio of Horizontal to Vertical Position, $\frac{x_f}{y_f}$**	**Angle of Incidence, θ_i (degrees)**
0	y_i =		$x_i = 0$	0	
50					
100					
150					
200					
250					
300					

**For m= 0 g, $y_i = y_f$. All other values of y_f are measured as the distance from y_i. Similarly, For m= 0 g, $x_i = x_f$. All other values of x_f are measured as the distance from x_i.*

5. On the provided graph (Figure 3), label the horizontal axis as "mass (m)" and the vertical axis as "Angle of Incidence θ_i (degrees)." Plot the angle of incidence as a function of added mass.

Relationship Between Angle of Incidence and Added Mass in a Cantilever

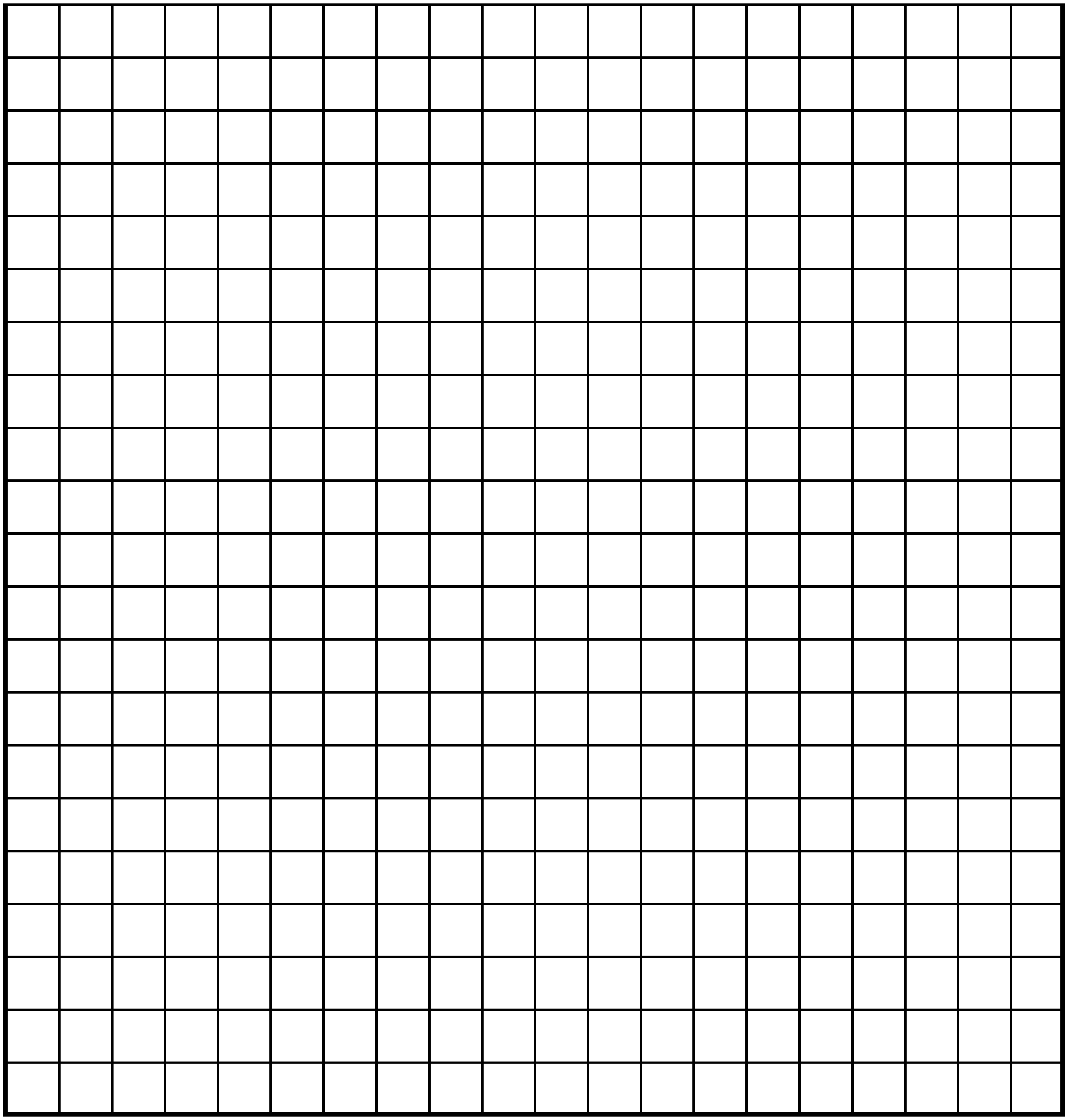


Figure 3. *Angle of Incidence versus Added Mass.*

6. Draw a line of best fit through the data points. What type of relationship best describes the graph you drew? What does the slope of the line of best fit represent? Describe the change in the angle of incidence as a function of added mass. Does this align with your trig equation from question 5 in the pre-lab? Why or why not?

7. On the provided graph (Figure 4), label the horizontal axis as "Angle of Incidence θ_i (degrees)" and the vertical axis as "Vertical Deflection Δy (cm)." Plot the vertical deflection as a function of the angle of incidence.

Relationship Between Deflection and Angle of Incidence in a Cantilever

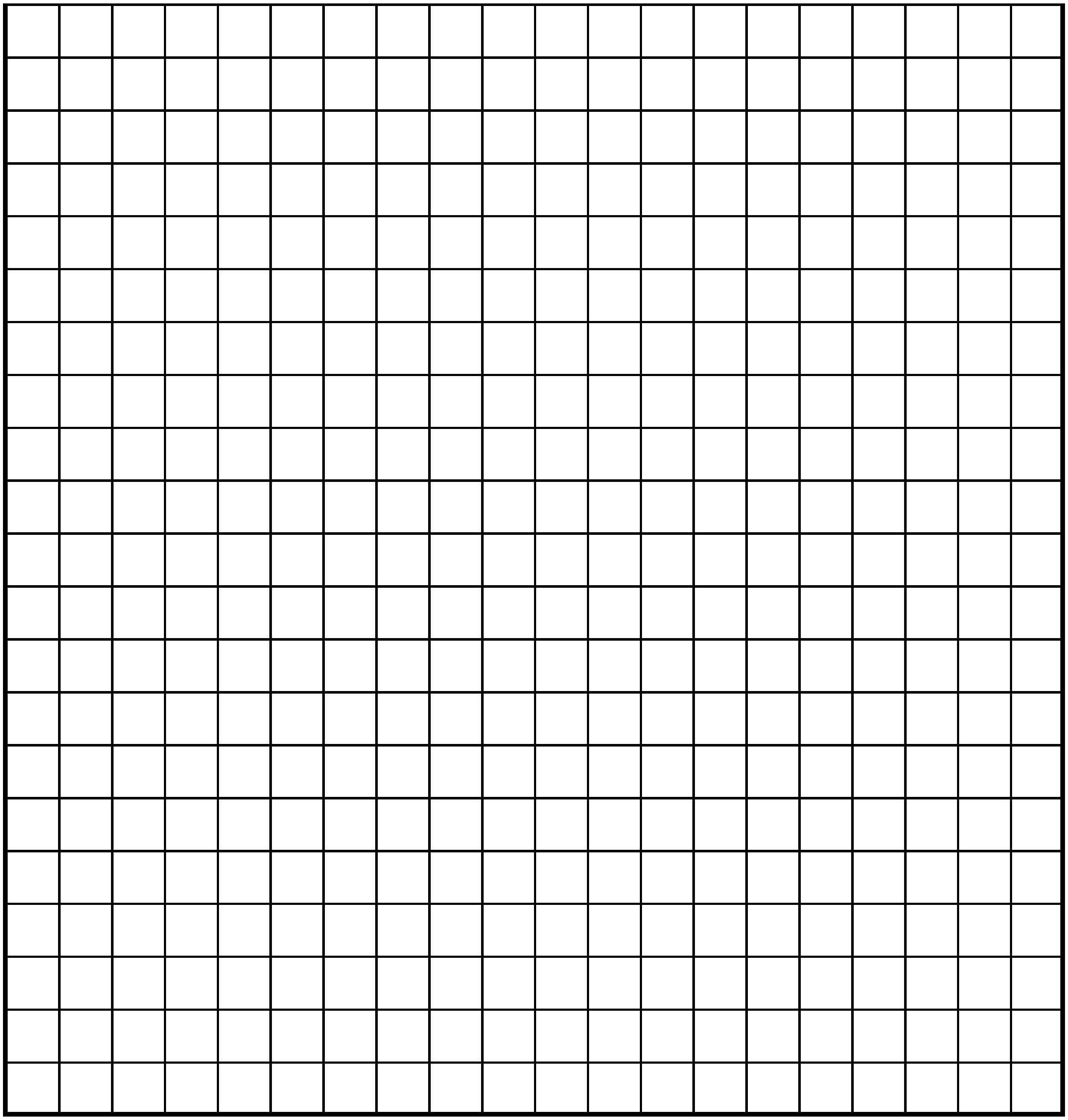


Figure 4. *Angle of Incidence versus Vertical Deflection.*

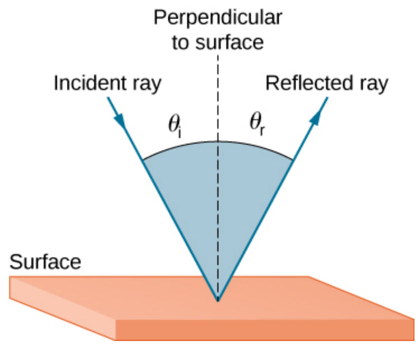
Assessment

1. Create a generalized formula that relates the angle of incidence and the deflection for the cantilever system in this investigation. Is this formula valid for all situations? Why or why not?

2. As the angle of incidence increases for a ray incident on a reflecting surface, what happens to the reflected angle? What must occur with a cantilever for the angle of incidence to increase?

3. What sources of error might there be with the experimental setup and instrumentation? Explain your thinking.

4. How does the law of incidence and reflection connect to real-life situations? Think back to the previous lessons. How does this relate to calculating Hooke's law and the usage of AFM?



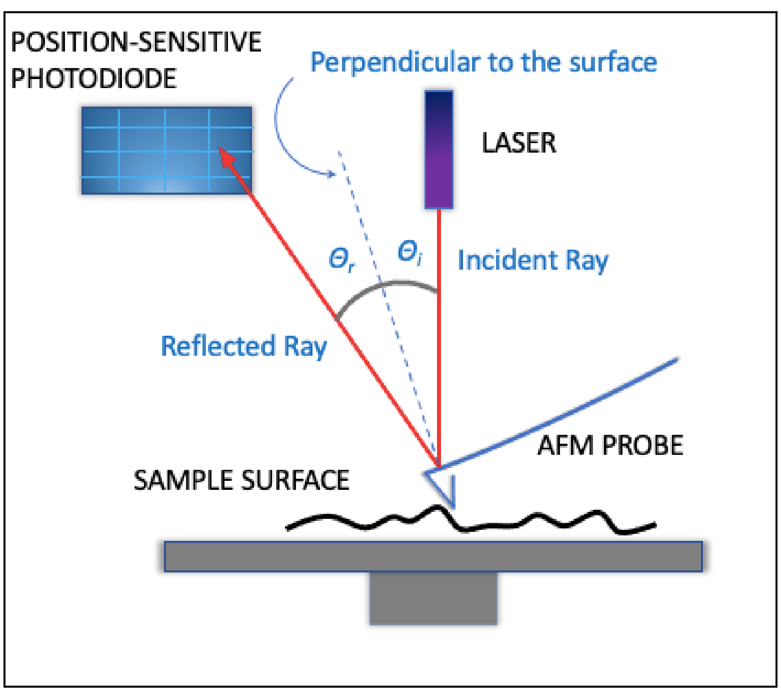
[Image Source](#)

Atomic Force Microscopes (AFMs)

PART 4: Statistical Analysis: Understanding How AMFs Use Optics & Reflected Light to Determine Sample Properties

Introduction

Atomic force microscopes (AFM) allow us to study samples at the nanoscale to measure and localize many forces, including adhesion, magnetic forces, and mechanical properties, to create a 3D surface profile of the sample. To do this, the AFM uses a cantilever system with a very sharp tip to traverse the surface of a material. Laser light is directed at a small mirror at the end of the cantilever. The **incident ray** then reflects onto a **position-sensitive photodiode (PSPD)**. As the cantilever deflects from forces exerted on the tip, slight changes occur in the **reflected ray**. The PSPD registers any fluctuations in the **angle of incidence** Θ_i and **angle of reflection** Θ_r. Thus, tracking the changes. As the AFM passes over a raised surface, the PSPD records the cantilever deflection and the resulting change in the angle of incidence.



Figures 1: *Diagram of the optics system of an AFM.*

From the equations derived in the last lesson, we can calculate (as a function of added mass) the expected values of horizontal deflection, vertical deflection, or angle of incidence. Then we can use statistical analysis to determine the effectiveness and significance of our instrumentation for mapping nanometer-sized surfaces.

In this lab, you will learn how to:

- identify and illustrate the path and angle of the reflected incident light.
- graphically represent the relationship between the change in the angle of the incident light and the mass added to the free end of a cantilever.
- graphically represent the relationship between the change in the angle of the incident light and deflection when mass is added to the free end of a cantilever.
- statistically analyze discrepancies between experimental values and theoretical values using a chi-squared test.
- draw a conclusion about the effectiveness of this experiment with statistical tests.

Activity: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

Safety Precautions:

- Mirrors are generally made of glass, so handle them carefully.
- Do not look directly into the laser; the light can damage your eye.
- Do not aim the laser at anyone's face.
- Keep the laser turned off when not taking measurements.
- Be mindful of your surroundings when using a meter stick to avoid collisions.
- Avoid putting excessive force on the meter stick and breaking it. If the meter stick breaks, be careful of the sharp end and dispose of it immediately.
- Be cautious when putting masses on the meter stick. Heavy masses can cause injury when dropped. Avoid standing or putting your hands in locations in which masses could fall.
- Be cautious when using the clamps so as not to pinch fingers.

For this activity, you will need the following:

- 2 wooden meter sticks
- 1 metric ruler
- 1" x 1" mirror
- 2 C-clamps
- laser pointer (red or green)
- ring-stand with 2 clamps (to hold the laser pointer and one of the meter sticks).
- weights (ranging in mass: 0 g, 50g, 100 g, 150g, 200 g, 250g, 300 g)
- weight holder or string
- two pieces of printer paper

- tape
- colored pencils & highlighters
- Protractor
- laptop or calculator (to do the χ^2 test and look up the χ^2 table for p-values)
- writing utensil

For this activity, you will work *in teams of three or four*.

Activity Introduction

During this activity, you will apply force on the free end of a cantilever attached to a table with a fixed clamp. Your cantilever will be a meter stick with a mirror attached to the underside of the free end. You will set up the laser pointer to shine vertically upward to the mirror. Your team will do five tests, with a different mass added to the end of the cantilever each time. Throughout the investigation, consider each trial independently and how the new trials compare to the previous ones.

Pre-Lab Questions

- First, carefully read the description and questions in the Procedure and the Data, Observation, & Analysis sections.
- Next, answer the pre-lab questions before starting the lab.

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1. What do you notice or wonder about the initial setup?

2. Considering the lab setup and your previous experience with cantilever systems. What are some sources of error? Explain your reasoning.

3. Do you predict the relationship between x_f, y_f, θ_i and the added mass to be the same as the last time? Explain your reasoning.

4. What are the intended takeaways from this activity?

Procedure

Step 1: Experimental Setup
Cantilever: Using the C-clamps, attach the meter stick horizontally to a table or desk surface with 70 cm of the meter stick hanging over the edge of the surface, as shown in Figure 2. Attach the mirror to the free end of the meter stick with 5 cm of free space at the end. Tie a length of string to the meter stick's free end. You can secure the string by wrapping it several times around the end of the meter stick and tapping it. Tape a piece of string of length 1 m to the top of the free end. Make sure the string aligns with the point where the laser hits the mirror.

Laser: Attach the laser pointer to the ring stand with a clamp, so the ray points vertically upward. *Use caution not to shine the laser at anyone or to look at it.* Place the ring stand directly under the mirror. Tape two pieces of paper together on the floor with the edge of one piece under the ring stand. You will record the position of the reflected beam on the paper as the lab progresses.

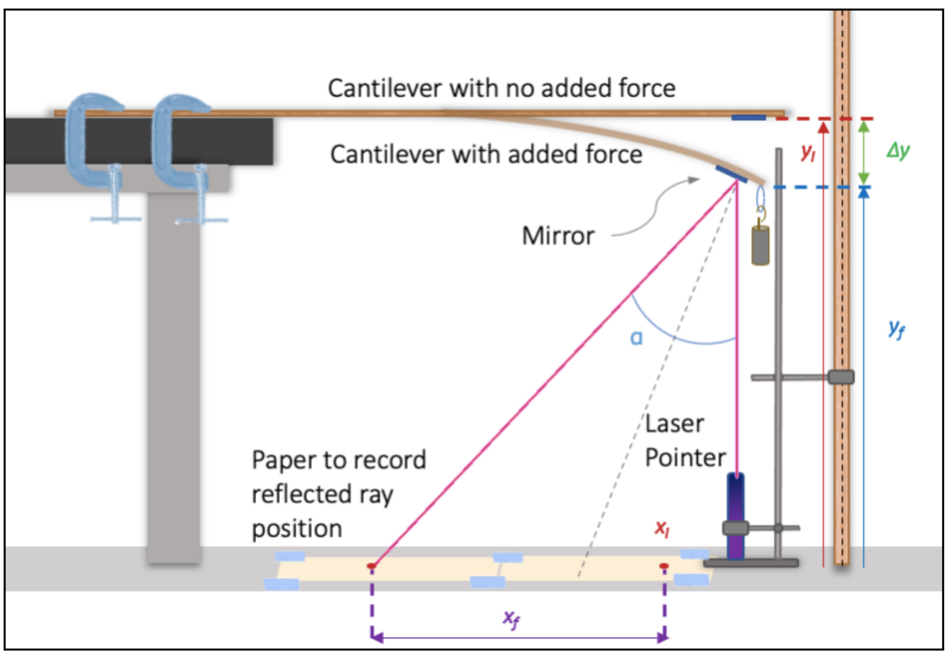

Figure 2: *Experimental setup*

Step 2: Take initial measurements

Cantilever height: Attach the second meter stick to the ring stand (perpendicular to the floor with the 0 cm at the floor.) Place the ring stand near the free end of the cantilever. Measure the height of the free end of the meter stick in centimeters to the closest millimeter. Record the initial height value in Table 1 as x_{i} (corresponding to 0 g of added mass).

Reflected light: With the laser pointer turned on, carefully mark where the reflected beam hits the paper. Label this point y_{i}. We will measure the distance the reflected beam hits the floor from this position as mass is added to the cantilever.

Step 3: Record the location of reflected light

Mark the location of the reflected ray on the paper. Measure the horizontal distance of the new reflected ray from the initial position x_i to the nearest millimeter and record it in centimeters in Table 1.

Step 4: Exert force on the cantilever, record new height & angle

Attach the 50 g mass to the string. Place the center of the protractor on the point y_{i}. Using the string, measure the angle α (where $\alpha = \Theta_{\mathrm{i}} + \Theta_{\mathrm{r}}$). Record the mass (in grams) and corresponding angle as α in Table 1. Measure the height of the free end of the meter stick, measuring upward from the floor. Record the mass (in grams) in Table 1.

Step 5: Repeat Steps 3 and 4 using 100g, 150g, 200g, 250g, and 300g.

Data, Observations, & Analysis

1. Estimate how much the cantilever deflected with no added mass. How about 100g of added mass?

2. As you work through the lab, for each trial:

    a. Record the mass *m* and the resulting height of the cantilever x_i in Table 1. Calculate the deflection Δy ($\Delta y = y_i - y_f$).

    b. Record the horizontal distance of the reflected ray relative to the initial position x_i.

    c. Calculate the ratio of the horizontal to the vertical position using x_i / y_i.

3. Using your string and protractor, measure angle α in degrees and record it in Table 1.

Table 1: Data Collection & Analysis

Mass of Attached Weight, *m* (g)	**Horizontal Position of the Reflected Ray, x_f (cm)***	**Vertical Height, y_f (cm)**	**Angle of Incidence, θ_i (degrees)**
0	$x_i = 0$	$y_i =$	90°
50			
100			
150			
200			
250			
300			

**For m= 0 g, $x_i = x_f$ and $\Theta_i = 90°$. All other values of x_f are measured as the distance from x_i and Θ_i is measured each time.*

4. Calculate the new height for each added mass using trig functions. Then find the deflection value by subtracting y_f from the initial height. Record your values in the following table:

Mass of Attached Weight, *m* (g)	**Angle of Incidence, θ_i (degrees)**	**Vertical Height Δy (cm)**	**Deflection $\Delta y = y_i - y_f$ (cm)**
0			
50			
100			
150			
200			
250			
300			

5. Do the vertical deflection values make intuitive sense? Explain your reasoning.

6. On the provided graph (Figure 3), label the horizontal axis as "Force (N)" and the vertical axis as "Vertical Deflection (cm)." Plot the angle of incidence as a function of added mass.

Relationship Between Force and Vertical Deflection

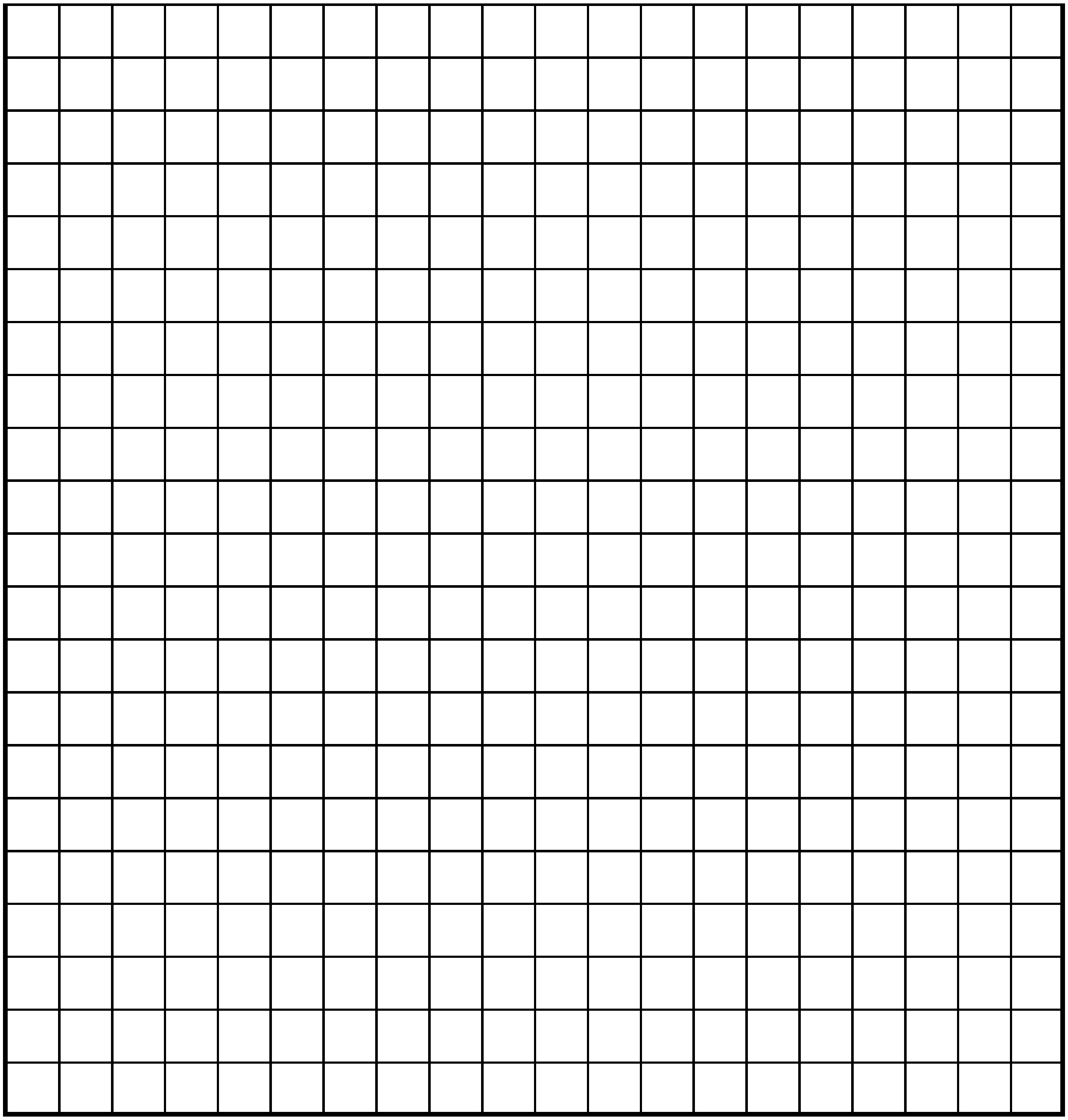

Figure 3. *Force versus Vertical Deflection.*

7. On the same graph (Figure 3), use a different color to plot the expected vertical deflections for a given force. How do the expected values compare with the observed values? Do your numbers make sense? Why or why not?

8. Using your trig equation from the previous lesson, find the expected angles and record them in the following table with the corresponding mass.

Mass of Attached Weight, *m* (g)	**Expected Angle of Incidence, θ_{Ei} (degrees)**
0	
50	
100	
150	
200	
250	
300	

9. Compare your expected angles with your measured angles. Do they seem similar, or are they different? If they were different, how could this impact the calculations for vertical deflection?

Statistical Analysis

We can use Person's Chi-Squared Test to determine if there is a statistically significant difference between the expected and observed values in one or more categories of a contingency table. To perform Person's Chi-Squared Test, we need the degrees of freedom. The degree of freedom is the least number of values you can know to solve for the Chi-Squared value. In this case, degrees of freedom, denoted as df, is equal to the sample space (denoted as n) minus one.

$$df = n - 1$$

A contingency table is a matrix that displays the multivariate frequency distribution of the variables. The following formula determines Person's Chi-Squared value:

$$\chi^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i}$$

10. In the following consistency tables, copy your expected and observed values.

Mass of Attached Weight, m (g)	Angle of Incidence, θ_i (degrees)	Expected Angle of Incidence, θ_{Ei} (degrees)
0		
50		
100		
150		
200		
250		
300		

11. Write your χ^2 value and degrees of freedom:
    a. χ^2 =
    b. df =

Assessment

1. Using a Chi-Squared Table or Chi-Squared calculator, state your p-value and significance level. What does your p-value tell you?
2. State your conclusion based on the results from the Chi-Squared Test.
3. According to your conclusion, would you recommend this instrumentation method for finding vertical deflection? Why or why not?
4. What changes could you make to better the experimental setup? What statistical tests could you run to make sure your experimental setup is an improvement?