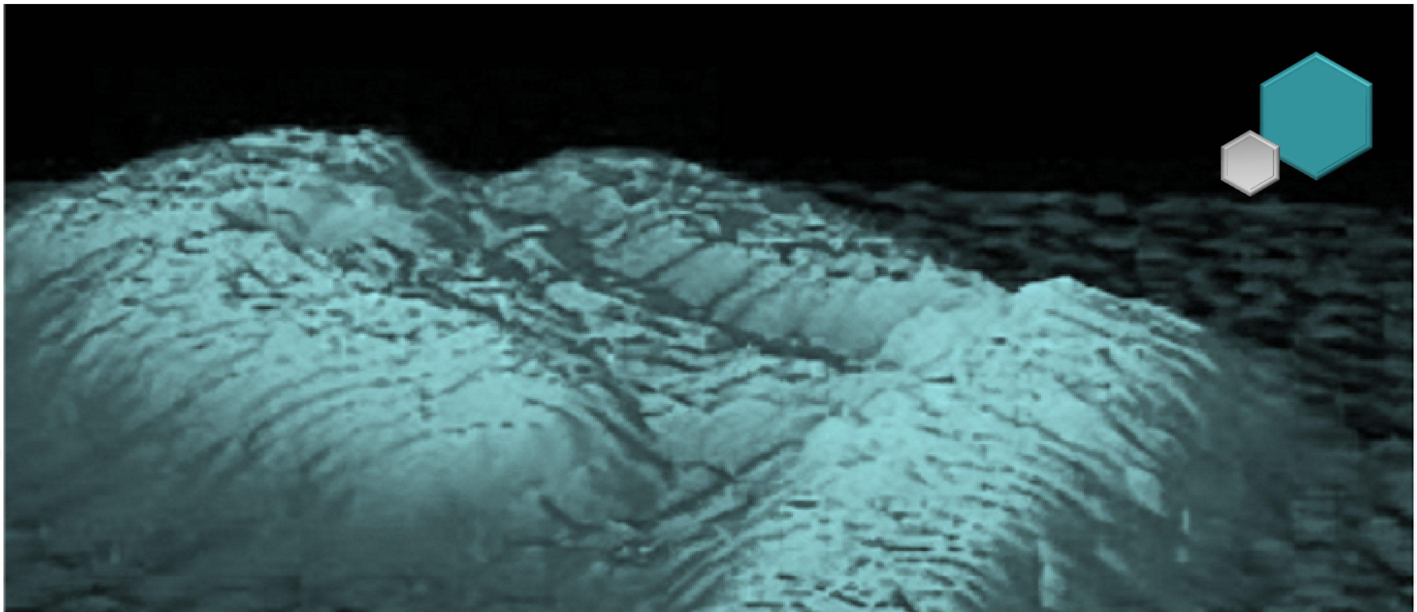
PHYSICS: Atomic Force Microscopes

How does an Atomic Force Microscope work?



PARTS 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

Image Credit: SAtomic Force Microscope Image, Eschericia coli--. C.J. Kazilek, Arizona State University. (2001). https://www.nsf.gov/news/mmg/mmg_disp.jsp?med_id=51835&from=search_list

Files Included for this Activity	5E Lesson Plans	Activity Lab Sheets*
PART 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers	√	√

*The activity Lab sheets are located at the end of this document.

Additional Files Available for this Series	5E Lesson Plans	Activity Lab Sheets*
PART 1: Understanding How Forces Impact Cantilevers & Springs	√	√
PART 3: Instrument Design: Understanding How AFMs Use Optics & Reflected Light to Determine Sample Properties	√	√
PART 4: Instrumentation Design: Statistical Analysis with Instrumentation	√	√

This module was developed by Professors Vernita Gordon and Alexandra Eusebi, with assistance from research & UTeach intern Khusbu R. Dalal, at the University of Texas, Austin, with funding provided by the National Science Foundation, Division of Civil, Mechanical, and Manufacturing Innovation, award numbers 1727544 and 2150878, to Vernita Gordon.

If you use any part of this module, please send an email describing your experience to Professor Gordon, gordon@chaos.utexas.edu. Please include the approximate number of students taught. Documenting this module's use and effectiveness will help us obtain more funding for outreach and education in the future.

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Driving questions for lesson: What are AFMs and how do they use forces to image surfaces on the nanometer scale?	
PART 1 Understanding How Forces Impact Cantilevers & Springs	• How do Atomic Force Microscopes use forces to map surfaces? • How do forces impact a cantilever's movements?
PART 2 Exploring Hooke's Law with the Deflection of Cantilevers	• How can we describe a cantilever system with Hooke's law? • How can we use mathematical models to describe a cantilever?
PART 3 Instrumentation Design: How AFMs Use Incident & Reflected Light	• How are angle of incidence and angle of reflection used to map a cantilever's deflection • Extension - How can we use statistical analysis to determine the effectiveness of a given experimental setup?
PART 4 Instrumentation Design: Statistical Analysis with Instrumentation	• How can we use statistical analysis to determine the effectiveness of a given experimental setup? • How do statistical tests help scientists determine the validity of an experiment and its results?

Texas Essential Knowledge and Skills (TEKS)

This lesson was developed as cross-curricular, supporting Physics and Mathematics TEKS.

Part 1	Part 2	Part 3	Part 4	§112.39. Physics
				(2) Scientific processes. The student uses a systematic approach to answer scientific laboratory and field investigative questions. The student is expected to:
√	√	√	√	(D) design and implement investigative procedures, including making observations, asking well defined questions, formulating testable hypotheses, identifying variables, selecting appropriate equipment and technology, evaluating numerical answers for reasonableness, and identifying causes and effects of uncertainties in measured data;
	√	√	√	(E) demonstrate the use of course apparatus, equipment, techniques, and procedures, including multimeters (current, voltage, resistance), balances, batteries, dynamics demonstration equipment, collision apparatus, lab masses, magnets, plane mirrors, convex lenses, stopwatches, trajectory apparatus, graph paper, magnetic compasses, protractors, metric rulers, spring scales, thermometers, slinky springs, and/or other equipment and materials that will produce the same results;
	√	√	√	(F) use a wide variety of additional course apparatus, equipment, techniques, materials, and procedures as appropriate such as ripple tank with wave generator, wave motion rope, tuning forks, hand-held visual spectroscopes, discharge tubes with power supply (H, He, Ne, Ar), electromagnetic spectrum charts, laser pointers, micrometer, caliper, computer, data acquisition probes, scientific calculators, graphing technology, electrostatic kits, electroscope, inclined plane, optics bench, optics kit, polarized film, prisms, pulley with table clamp, motion detectors, photogates, friction blocks, ballistic carts or equivalent, resonance tube, stroboscope, resistors, copper wire, switches, iron filings, and/or other equipment and materials that will produce the same results;
	√	√	√	(G) make measurements with accuracy and precision and record data using scientific notation and International System (SI) units;
	√	√	√	(H) organize, evaluate, and make inferences from data, including the use of tables, charts, and graphs;
√	√	√	√	(I) communicate valid conclusions supported by the data through various methods such as lab reports, labeled drawings, graphic organizers, journals, summaries, oral reports, and technology-based reports; and
	√	√	√	(J) express relationships among physical variables quantitatively, including the use of graphs, charts, and equations.
				(3) Scientific processes. The student uses critical thinking, scientific reasoning, and problem solving to make informed decisions within and outside the classroom. The student is expected to:
√	√	√	√	(A) analyze, evaluate, and critique scientific explanations by using empirical evidence, logical reasoning, and experimental and observational testing, so as to encourage critical thinking by the student;
				(4) Science concepts. The student knows and applies the laws governing motion in a variety of situations. The student is expected to:
	√	√		(D) calculate the effect of forces on objects, including the law of inertia, the relationship between force and acceleration, and the nature of force pairs between objects using methods, including free body force diagrams.

Part 1	Part 2	Part 3	Part 4	§111.39. Algebra I
				(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:
	√	√	√	(B) use a problem-solving model that incorporates analyzing given information, formulating a plan or strategy, determining a solution, justifying the solution, and evaluating the problem-solving process and the reasonableness of the solution;
	√	√	√	(C) select tools, including real objects, manipulatives, paper and pencil, and technology as appropriate, and techniques, including mental math, estimation, and number sense as appropriate, to solve problems;
	√	√	√	(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
	√	√	√	(E) create and use representations to organize, record, and communicate mathematical ideas;
	√	√	√	(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
	√	√	√	(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
				(2) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using properties of linear functions to write and represent in multiple ways, with and without technology, linear equations, inequalities, and systems of equations. The student is expected to:
	√	√	√	(B) write linear equations in two variables in various forms, including y = mx + b, Ax + By = C, and y – y1 = m(x – x1), given one point and the slope and given two points;
	√	√	√	(C) write linear equations in two variables given a table of values, a graph, and a verbal description;
				(3) Linear functions, equations, and inequalities. The student applies the mathematical process standards when using graphs of linear functions, key features, and related transformations to represent in multiple ways and solve, with and without technology, equations, inequalities, and systems of equations. The student is expected to:
	√	√	√	(A) determine the slope of a line given a table of values, a graph, two points on the line, and an equation written in various forms, including y = mx + b, Ax + By = C, and y – y1 = m(x – x1);
	√	√	√	(C) graph linear functions on the coordinate plane and identify key features, including x-intercept, y-intercept, zeros, and slope, in mathematical and real-world problems;
				(4) Linear functions, equations, and inequalities. The student applies the mathematical process standards to formulate statistical relationships and evaluate their reasonableness based on real-world data. The student is expected to:
			√	(A) calculate, using technology, the correlation coefficient between two quantitative variables and interpret this quantity as a measure of the strength of the linear association;
	√	√	√	(C) write, with and without technology, linear functions that provide a reasonable fit to data to estimate solutions and make predictions for real-world problems.
	√	√	√	(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
	√	√	√	(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.

Part 1	Part 2	Part 3	Part 4		§111.4. Geometry
				(1) Mathematical process standards. The student uses mathematical processes to acquire and demonstrate mathematical understanding. The student is expected to:	
√	√	√	√		(D) communicate mathematical ideas, reasoning, and their implications using multiple representations, including symbols, diagrams, graphs, and language as appropriate;
√	√	√	√		(E) create and use representations to organize, record, and communicate mathematical ideas;
√	√	√	√		(F) analyze mathematical relationships to connect and communicate mathematical ideas; and
√	√	√	√		(G) display, explain, and justify mathematical ideas and arguments using precise mathematical language in written or oral communication.
				(7) Similarity, proof, and trigonometry. The student uses the process skills in applying similarity to solve problems. The student is expected to:	
		√	√		(B) apply the Angle-Angle criterion to verify similar triangles and apply the proportionality of the corresponding sides to solve problems.
				(9) Similarity, proof, and trigonometry. The student uses the process skills to understand and apply relationships in right triangles. The student is expected to:	
		√	√		(A) determine the lengths of sides and measures of angles in a right triangle by applying the trigonometric ratios sine, cosine, and tangent to solve problems

<table>
<tr><th>Objective/s- Write objective/s
The student should be able to:</th><th>Assessment:
What will you accept as evidence of student progress toward your lesson objective?</th></tr>
<tr><td>PART 1

Identify the forces acting upon a cantilever.
Demonstrate and describe qualitatively how force causes deflection of a cantilever.
Describe qualitatively how a material's adhesive properties impact the force required to detach it from a surface.
</td><td>Part 1

Students will articulate how the lab setup models the real-life scenario of AFMs and the mechanical force/action required to free the cantilever. Students will explain how the movement of their arm exerts a force on the model. Students will also be able to qualitatively describe and order the force required to disturb various tape samples and relate these forces to the cantilever's deflection.
Formative Assessment: The teacher will formatively assess student understanding as they walk around and ask questions to students/groups as they are working through the activity or handout.
</td></tr>
<tr><td>PART 2

Identify and illustrate the forces acting upon a mass suspended from a cantilever by drawing and labeling a free body diagram for the system in equilibrium.
Graphically represent the relationship between deflection and mass added to the free end of a cantilever.
Mathematically model the behavior of a cantilever system with an equation and graph, developing Hooke's law and recognizing the spring constant.
</td><td>PART 2

Students will be able to quantitatively describe how a cantilever's deflection relates to the amount of force exerted with the use of Hooke's law.
Students will calculate the spring constant for the given cantilever system.
Students will algebraically manipulate (rearrange) the equation for the spring constant.
Students should be able to draw a free body diagram for the mass suspended from a cantilever.
Formative Assessment will be happening as the teachers walk around and ask questions to students/groups as they are working through the activity or handout.
</td></tr>
</table>

<table>
<tr><th>Objective/s- Write objective/s
The student should be able to:</th><th>Assessment:
What will you accept as evidence of student progress toward your lesson objective?</th></tr>
<tr><td>PART 3
Identify and illustrate the path and angle of the reflected incident light.
Graphically represent the relationship between the change in the angle of the incident light and the mass added to the free end of a cantilever.
Graphically represent the relationship between the change in the angle of the incident light and deflection when mass is added to the free end of a cantilever.</td><td>PART 3
Students will effectively articulate how instrumentation with angle of reflection and angle of incidence can be used to map surfaces with AFMs.
Formative Assessment will be happening as the teachers walk around and ask questions to students/groups as they are working through activity or handout.</td></tr>
<tr><td>PART 4
Identify and illustrate the path and angle of the reflected incident light.
Graphically represent the relationship between the change in the angle of the incident light and the mass added to the free end of a cantilever.
Graphically represent the relationship between the change in the angle of the incident light and deflection when mass is added to the free end of a cantilever.
Statistically analyze discrepancies between experimental values and theoretical values using a chi-squared test.
Draw a conclusion about the effectiveness of this experiment with statistical tests.</td><td>PART 4
Students will be able to determine the effectiveness of this instrumentation through the use of statistical analysis (Pearson’s Chi-Squared Test).
Students will calculate the p-value for Pearson’s Chi-Squared Test through the use of excel or other computer software and draw a conclusion based on that finding.
Formative Assessment will be happening as the teachers walk around and ask questions to students/groups as they are working through activity or handout.</td></tr>
</table>

PART 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

ENGAGEMENT

Estimated Time: 10 minutes

Overview of Activity: Students will be introduced to deflection and will relate it to an everyday activity. In the process, the students will connect the deflection of a cantilever to how an AFM functions to quantify characteristics of a sample.

Resources Needed: whiteboard & dry erase markers or poster paper & markers to record student comments

Safety Considerations: None.

What the teacher is doing:	What the student does:	Possible questions to ask students and anticipated student responses:
The teacher starts the lesson by asking students about their favorite summertime activities. The teacher then asks specifically about swimming and diving. Show a video on diving. After students have a chance to respond, relate the diving board to a cantilever. The more force the diver puts on the cantilever (diving board), the further it deflects. Mention how this is today's experiment with a meter stick as a cantilever and how this relates to AFMs; further discuss the relation to Hooke's law. The teacher introduces the idea of Atomic Force Microscopes (AFM) and how the deflection of the cantilever can be represented by Hooke's law. The teacher instructs students to do the pre-lab questions independently	Students share their responses. If desired, or if prompting is needed, students can do this think-pair-share style: students think about the question independently, discuss possible answers with a partner, and then share with the class.	How many of you like swimming? (This can also be related to the Olympic Diving if it is a competition year.) Have any of you been diving? What do you know about the mechanics of a diving board? • *They bend when you jump on them.* How do you think understanding the mechanics of a diving board would help scientists develop equipment? • *Some equipment might bend like a diving board does?* Have you ever heard of Hooke's Law? What do you know about it? • *Never heard of it.* • *Has to do with spring.*

for 5 minutes, then they can discuss with their groups. The teacher prepares a large page or whiteboard to write students' responses. The teacher facilitates students' discussion and invites groups to share their answers.		

PART 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

EXPLORATION

Estimated Time: 25 minutes

Overview of Activity: Students will explore Hooke's law through experimentation and graphing the relationship of force as a function of displacement. In the process, students will quantitatively evaluate the forces within the system and calculate the spring constant for their cantilever.

Resources Needed:

- Per group: 2 wooden meter sticks, 2 C-clamps, masses ranging in 0 g, 50g, 100 g, 150g, 200 g, 250g, 300 g, mass holder or string, tape
- Per student: "AFM Activity 2 Lab Sheet – Calculating Hooke's Law" handout, writing utensils
- Note: You may also want to test the equipment prior to completing the activity with students to ensure you are not asking students to add more mass to the system than the meter stick can hold.

Safety Considerations: Be mindful of your surroundings when using a meter stick to avoid collisions. Avoid putting excessive force on the meter stick and breaking it. If the meter stick breaks, be careful of the sharp end and dispose of it immediately. Be cautious when putting weights on the meter stick. Heavy weights can cause injury when dropped. Avoid standing or putting your hands in locations in which weights could fall. Be cautious when using the clamps so as not to pinch fingers.

What the teacher is doing:	What the student does:	Possible questions to ask students and anticipated student response:
Throughout the activity, the teacher takes the role of facilitator and "lead-learner." Explain instructions and expectations to students. Students will begin the activity portion of the lesson by reading through the entire handout. Pass out materials to each student group or have a predetermined team equipment manager pick them up from a designated location. Pass out the "AFM Activity 2 Lab Sheet – Calculating Hooke's Law" handout to each student.	In student groups of 3-4: Students read through the "Procedure" section of the "AFM Activity 2 Lab Sheet – Calculating Hooke's Law" handout. Students will discuss the activity and observation questions with their group, recording what they know and wonder about and discussing any potential questions or concerns. Students will follow the directions on the handout to set up the experiment before	What do you notice about the meter stick without any mass attached? • *The meter stick is sagging down.* • *The meter stick is drooping.* How could having a bent meter stick initially affect your data? • *It could make the initial data point inaccurate.* As you are adding mass, what does your new height look like? Is it constantly changing? • *The new height is getting smaller.*

After students read through the handout, reiterate instructions and have students discuss the activity with their group and develop a short list of what they know and wonder. Have the students think through their questions and try to reason through them before sharing them with you. Doing so will identify student groups uncertain of the directions and promote cooperation skills while encouraging autonomy. Note: It may be necessary to model how to set up the experiment before passing out materials. As students work in their groups, ● Make sure each group has set up the experiment correctly. ● Ensure each student is participating. ● Rotate between groups and ask guiding questions to individual students within each group: ○ Goal: provide students time to process the different aspects of the system and how one may impact the other and assess student understanding as they work through the activity. ○ Use open-ended and guiding questions to encourage critical thinking while subsequently holding students accountable for their learning.	proceeding to conduct the experiment and recording their findings. Once completed, students will disassemble the experiment and return the materials to the proper location.	When you take off the mass, what happens to the meterstick? Why? ● *The meter stick bounces back up because the system wants to go back to equilibrium.* How much mass do you think we can add before the cantilever breaks? ● *1000 grams* What would your data look like if you were to add mass until the cantilever breaks? ● *The force versus displacement would start out as a linear relationship, but would start looking like a nonlinear graph with more added mass.*

PART 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

EXPLANATION

Estimated Time: 15 minutes

Overview of Activity: Students will mathematically model the behavior of a cantilever system with an equation and graph, developing Hooke's law and recognizing the spring constant.

Resources Needed: handout, document camera.

Safety Considerations: None.

What the teacher is doing:	What the student does:	Possible questions to ask students and anticipated student response:
The teacher guides a whole-group discussion over the students' experiences during the activity and their observations. The teacher asks students to summarize the activity and their observations. The teacher acts as the facilitator and provides prompting, guiding questions, and open-ended questions. The teacher records student comments on the white board or poster paper. The teacher allows students to present their ideas and findings to the class on the board/document camera. • Take this opportunity to ask the class if they got the same/different answers. This provides a chance for further	Students follow the teacher's lead in a whole group discussion. Students discuss: • The relationship between the experiment's setup and how it models Hooke's law. • The relationship between applied force and displacement. Another option would be to have each group create a poster summarizing the experiment and the group's observations. Student groups would present their posters and answer guiding questions provided by the teacher. • Could also do this in a jigsaw or gallery walk, which allows the students to provide anonymous feedback.	*NOTE: For additional guiding questions, see the suggested questions in the exploration phase* What is the overall relationship between the force and deflection? • *There is a linear relationship between the force and deflection.* Why is it negatively linear? • *The displacement $\Delta x = x_i - x_f$, but if we were to write it as the traditional $\Delta x = x_f - x_i$, the negative would come out.* • *As the force increases, the free end (height) of the cantilever is getting smaller.* Do all the points go through the line of best fit? Why or why not might this be? • *Not all the points go through the line of best fit. This could be due to random error (from measurement taking) or systematic error (the meterstick already sagging).*

discussion and includes all of the class.		What could we do differently in our experimental set up that could help remedy this? • *Ensure the cantilever is perfectly horizontal with 0g added mass.* • *Use more precise measuring tools.*

PART 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

ELABORATION

Estimated Time: 15 minutes

Overview of Activity: Provides students an opportunity to apply or extend the new ideas and information on the exploration of Hooke's law through a cantilever system.

Resources Needed: whiteboard & dry erase markers or poster paper and markers to record student comments

Safety Considerations: None.

What the teacher is doing:	**What the student does:**	**Possible questions to ask students and anticipated student response:**
The teacher facilitates the discussion by providing guiding questions geared to help students apply or expand upon the day's lessons over cantilever systems and Hooke's law. The teacher records student comments on the whiteboard or poster paper. The teacher asks students to consider how you could quantify adhesion bond strength. The teacher guides students to discuss the various forces that act upon the system and how the cantilever system models Hooke's law.	Students take part in active discussion and subsequent activity. If desired, or if prompting is needed, students can do this think-pair-share style: students think about the question independently, discuss possible answers with a partner, and then share with the class. Students draw free body force diagrams depicting the lab activity.	*Questions will depend on the direction the elaboration phase takes. See Extension activity. Possible ideas:* Is there a point where Hooke's law would fail? Why might this be? • *There is not a point where Hooke's law would fail since it's a linear equation. (This is incorrect)* • *Hooke's law will fail if the spring or cantilever is loose or overstretched. When taking data with flawed equipment, we will see that it does not follow Hooke's law.* If a group got a curved line, this could mean that the meter stick was bent beyond its normal range. In that case, Hooke's law would fail. Compare spring contents with other groups and materials.

PART 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

EVALUATION

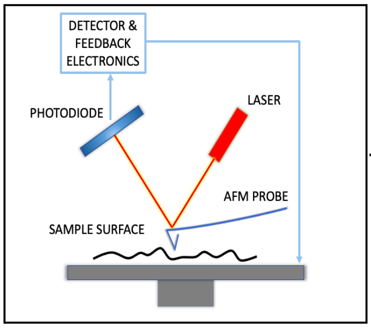
Estimated Time: Embedded throughout the lesson. Account for time required for pre-assessments and post-assessments (if necessary).

Overview of Activity: Formative and summative assessment of student understanding and effectiveness of the lesson.

Resources Needed: varies

Safety Considerations: None.

What the teacher is doing:	What the student does:	Possible questions to ask students and anticipated student response:
Throughout the lesson and activity, the teacher uses formative assessments to determine how much the students understand and if they are grasping the new concepts, as well as to evaluate the quality of the lesson. Formative assessment occurs during the learning process, provides an opportunity to identify the need for flexibility in instruction, identifies areas for improvement (both for student and for the lesson), and allows students a chance to implement feedback. Examples of formative assessment include hand signals, direct or indirect questioning, quizzes, observations, homework, and classwork. Summative assessment is completed at the end of the learning process and provides an evaluation of student concept knowledge. Summative assessment will often include evaluating the student work products, such as responses on the guided notes/handouts, posters, teacher-created exit slips, or unit tests.	Throughout the lesson, students work in groups to answer questions regarding the experiment and observations. Students will also answer questions independently when prompted by the teacher during the lesson phases or through teacher-prepared pre-assessments, post-assessments, or exit slips.	*See questions embedded in the sections above. Also, see accompanying Part 3 handout.*



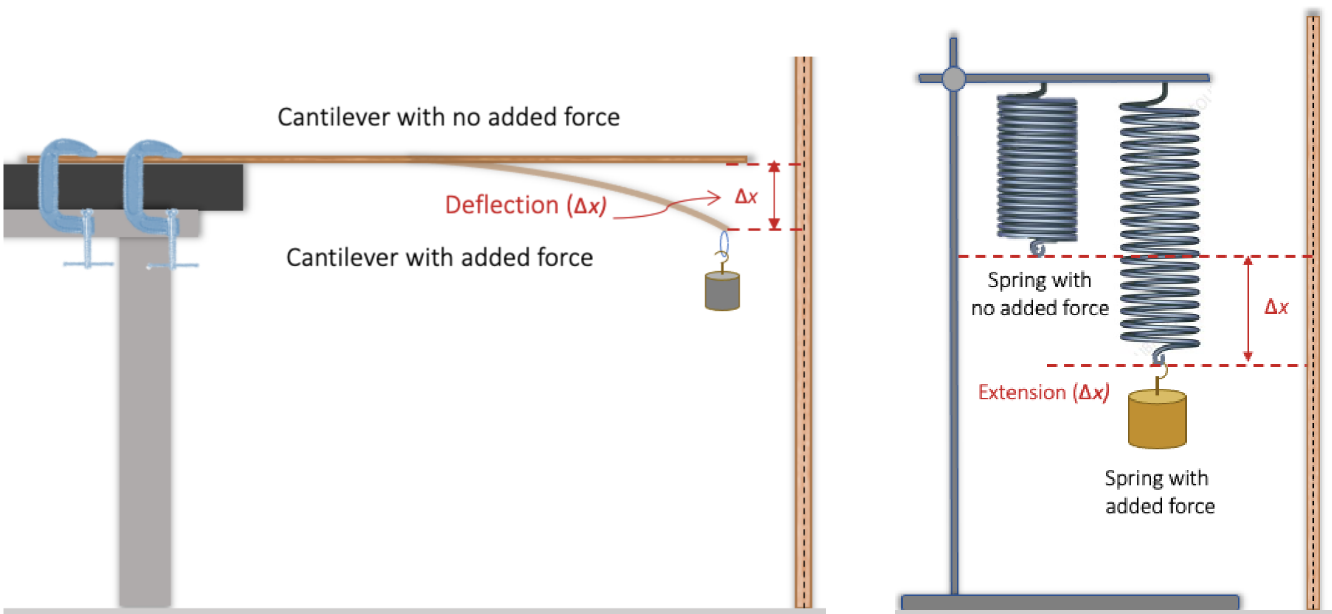
Atomic Force Microscopes (AFMs)

PART 2: Hooke's Law: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

Introduction

Atomic force microscopes (AFM) use a cantilever to measure and localize many forces, including adhesion, magnetic forces, and mechanical properties, to create a 3D surface profile of the sample. Much like a spring, the magnitude of the cantilever's deflection, or displacement, depends on the beam's material and the force applied to it.

With cantilevers, **Hooke's law** mathematically relates the **deflection** (Δx) of a cantilever when weight is added to the **force** (F) exerted by the cantilever to keep the system in equilibrium. Similarly, with springs, Hooke's law relates the force applied to an unstretched spring and the amount it stretches. Although some materials may seem similar, each system exhibits Hooke's law differently due to an inherent characteristic called the **spring constant** (k). Figures 1a & 1b show Hooke's law for cantilever and spring in action, respectively.



Figures 1a & 1b: *Displacement of a cantilever and spring due to application of mass on the system.*

We can validate Hooke's law by adding mass to a cantilever system and measuring the resulting deflection. This activity will provide insight into how an AFM works by quantitatively investigating the impact of force applied to a cantilever and deriving Hooke's law for a given system.

In this lab, you will learn how to:

- Identify and illustrate the forces acting upon a mass suspended from a cantilever by drawing and labeling a free body diagram for the system in equilibrium.
- Graphically represent the relationship between deflection and mass added to the free end of a cantilever.
- Mathematically model the behavior of a cantilever system with an equation and graph, developing Hooke's law and recognizing the spring constant.

Activity: Calculating the Relationship Between Deflection & Applied Force in Cantilevers

Safety Precautions:

- Be mindful of your surroundings when using a meter stick to avoid collisions.
- Avoid putting excessive force on the meter stick and breaking it. If the meter stick breaks, be careful of the sharp end and dispose of it immediately.
- Be cautious when putting weights on the meter stick. Heavy weights can cause injury when dropped. Avoid standing or putting your hands in locations in which weights could fall.
- Be cautious when using the clamps so as not to pinch fingers.

For this activity, you will need the following:

- 2 wooden meter sticks
- 2 C-clamps
- weights (ranging in mass: 0 g, 50g, 100 g, 150g, 200 g, 250g, 300 g)
- weight holder or string
- tape
- writing utensil

For this activity you will work *in teams of three or four*.

Activity Introduction

During this activity, you will apply force on the free end of a cantilever attached to a table with a fixed clamp. Your cantilever will be a meter stick. Your team will conduct the experiment five times, using different masses for each attempt. Throughout the investigation, consider each trial independently and how the new trials compare to the previous ones.

Pre-Lab Questions

- First, carefully read the description and questions in the Procedure and the Data, Observation, & Analysis sections.
- Next, answer the pre-lab questions before starting the lab.

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1. What do you notice or wonder about the initial setup?

2. What forces are acting upon the cantilever shown in Figure 2 (before adding mass to the meter stick)? What forces are acting upon the mass when added to the free end of the meter stick?

3. What do you anticipate will happen to the meter stick when you apply force to the free end of the meter stick? Explain your thinking.

4. In this lab, you will derive Hooke’s law – the mathematical relationship between force exerted upon a cantilever or spring and the resulting displacement. What mathematical relationship do you hypothesize would best describe this relationship (ex., proportional, inversely related, quadratic)? Explain your thinking.

5. What do you hypothesize will happen to the cantilever when we remove the mass from the system? Explain your thinking.

6. What are the intended takeaways from this activity?

Procedure

Step 1: Using the C-clamps, attach the meter stick horizontally to a table or desk surface with 70 cm of the meter stick hanging over the edge of the surface, as shown in Figure 2.

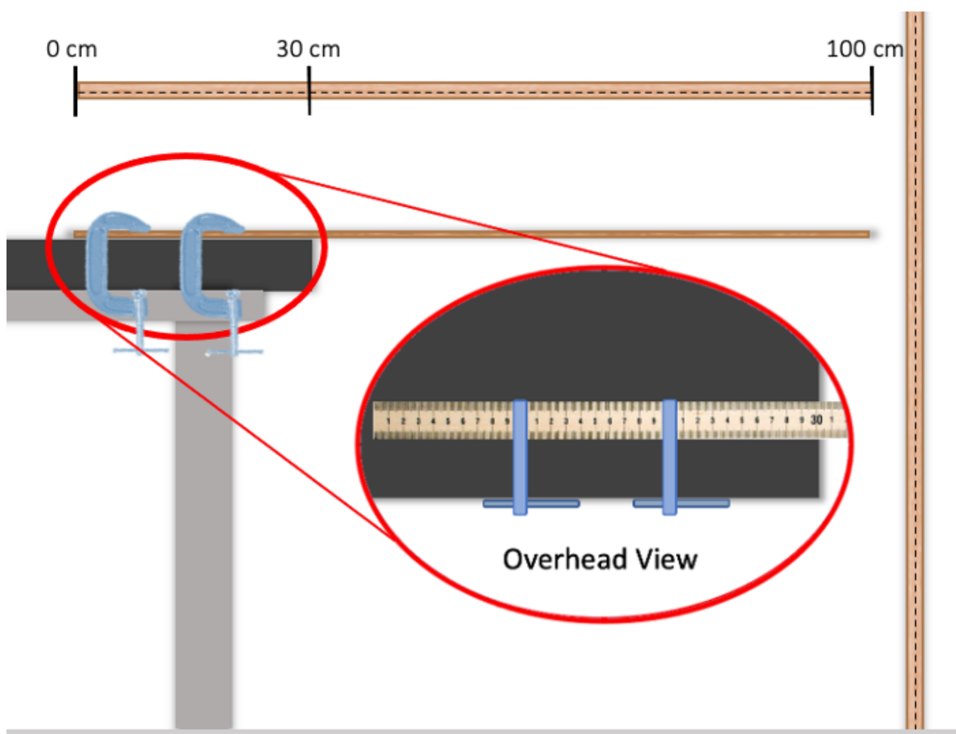

Figure 2: *Experimental setup*

Step 2: Place the second meter stick near the cantilever (perpendicular to the floor with the 0 cm at the floor.) Measure the height of the free end of the meter stick in centimeters to the closest millimeter. Record the initial height value in Table 1 as x_i (corresponding to 0 g of added mass).

Step 3: Tie a length of string to the meter stick's free end. You can secure the string by wrapping it several times around the end of the meter stick and tapping it.

Step 4: Attach the 100 g weight to the string and measure the height of the free end of the meter stick, measuring upward from the floor. Record the mass of the added weight (in grams) and corresponding final height x_f of the end of the meter stick in Table 1.

Step 5: Repeat Step 4 using 100 g, 150 g, 200 g, 250 g, and 300 g.

Data, Observations, & Analysis

7. Draw and label two free body diagrams of the mass: one while the mass is not attached to the cantilever and the second with the mass suspended from the cantilever. Be sure to include the force of weight.

8. As you work through the lab, record the mass of the attached weights and the resulting height of the cantilever in Table 1. Calculate the vertical deflection by subtracting the new height of the cantilever's end after each trial from the initial height ($\Delta x = x_i - x_f$).

Table 1: Data Collection

Mass of Attached Weight, *m* (g)	Vertical Height, x_f (cm)	Vertical Deflection, $\Delta x = x_i - x_f$ (cm)*
0	x_i =	0
100		
200		
300		
400		
500		

**For m= 0 g, $x_i = x_f$,therefore, Δx = 0 cm.*

9. Calculate the force (F) exerted on the cantilever by the added masses for each trial. Record the values in Table 2. Recall that force is equal to mass times acceleration (F = m·a), where a is the acceleration due to gravity (a = 9.81 m/s^2). Copy the vertical deflection from Table 1 to Table 2, **converting centimeters to meters**. Note that the mass units have been converted to kg to report force in terms of Newtons (N = kg·m/s^2).

Table 2: Data Processing

Mass of Attached Weight, m (kg)	Force Exerted by Cantilever on the weight, F = m·a (N)	Vertical Deflection, $\Delta x = x_i - x_f$ (m)*
0.000	0	0
0.100		
0.200		
0.300		
0.400		
0.500		

**Report Δx in meters.*

10. On the provided graph (Figure 3), label the x-axis as "Vertical Deflection (Δx)" and the y-axis as "Force (F)." Plot the deflection as a function of added mass. Refer to the deflection distance using the symbol Δx.

Modeling Hooke's Law

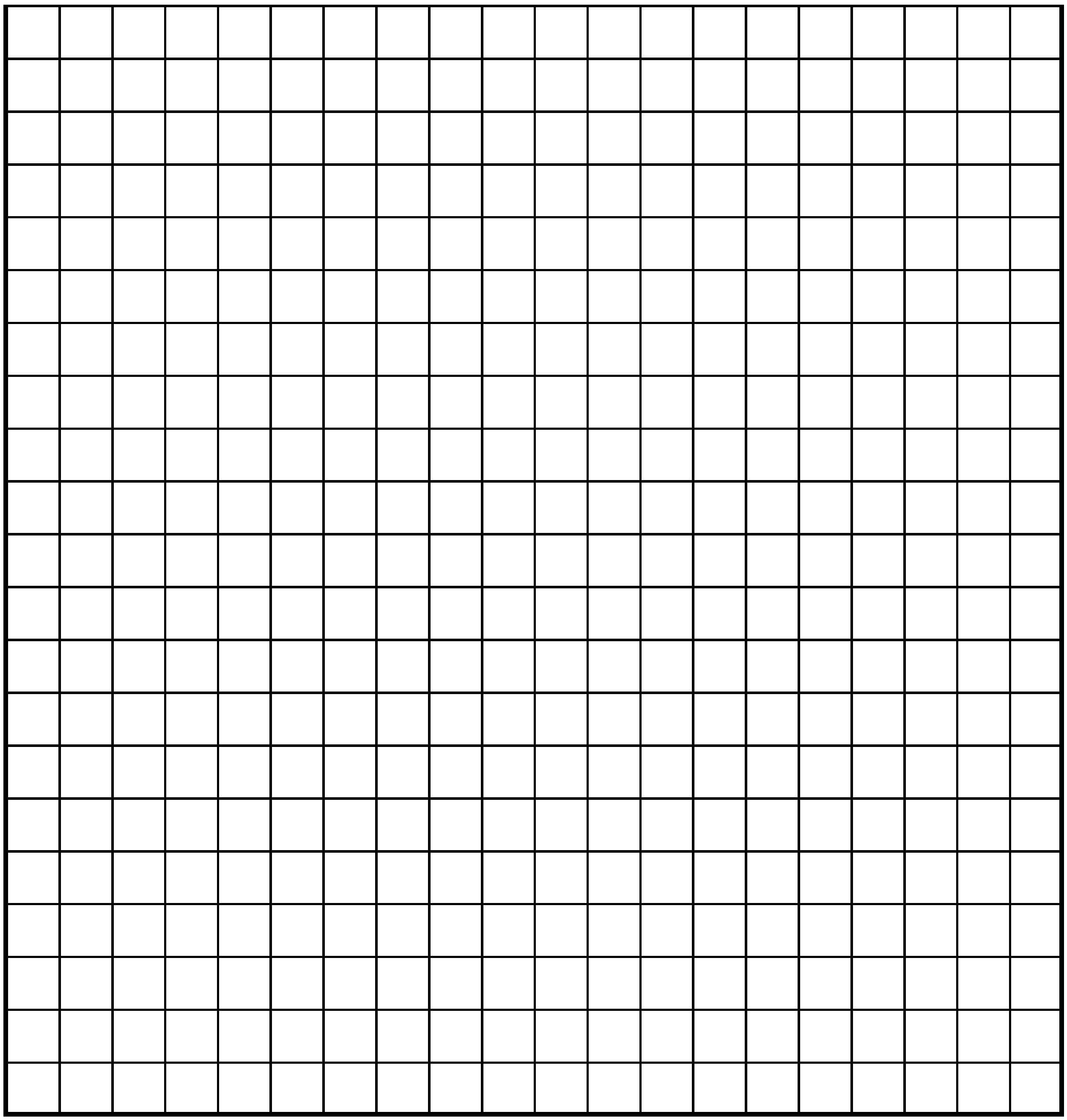

Figure 3. *Force versus Vertical Deflection*

11. Draw a line of best fit through the data points. What type of relationship best describes the graph you drew?

12. Determine the slope of the line of best fit from your graph. Be sure to include units. Give this slope the symbol k. How does your k value compare to those found by groups around you? What does the slope of the line of best fit represent? Use the units of the slope in the description of its meaning. Relate the meaning of the slope to this particular lab and meter stick. Use your specific slope as an example.

13. Determine an equation that best represents the function shown on your graph. Be sure to use the symbols F and Δx. What is your y-intercept, and what does it mean? Should your equation have a y-intercept of 0 Newtons? Explain why or why not.

Extension Activities

1. Consider what would happen if you used a different cantilever. How would the results change if the meter stick was thinner or thicker? What if you used a cantilever with a completely different shape, such as a dowel rod? How would you expect the results for a metal meter stick to differ?

   Repeat the experiment with a different cantilever and hypothesize how the results would compare to the results for the original experiment. After completing the experiment, comment on if your hypothesis was correct or incorrect. Explain your answer.

   Table 3: Extension – Data Collection

Mass of Attached Weight, *m* (g)	Vertical Height, x_f (cm)	Vertical Deflection, $\Delta x = x_i - x_f$ (cm)*
0	x_i =	0
100		
200		
300		
400		
500		

**For m= 0 g, x_i=x_f ,therefore, Δx = 0 cm.*

Table 2: Extension – Data Processing

Mass of Attached Weight, m (kg)	Force Exerted by Cantilever on the weight, F = m·a (N)	Vertical Deflection, $\Delta x = x_i - x_f$ (m)*
0.000	0	0
0.100		
0.200		
0.300		
0.400		
0.500		

**Report Δx in meters.*

2. On the provided graph (Figure 4), label the x-axis as "Vertical Deflection (Δx)" and the y-axis as "Force (F)." Plot the deflection as a function of added mass. Refer to the deflection distance using the symbol Δx.

Extension - Modeling Hooke's Law

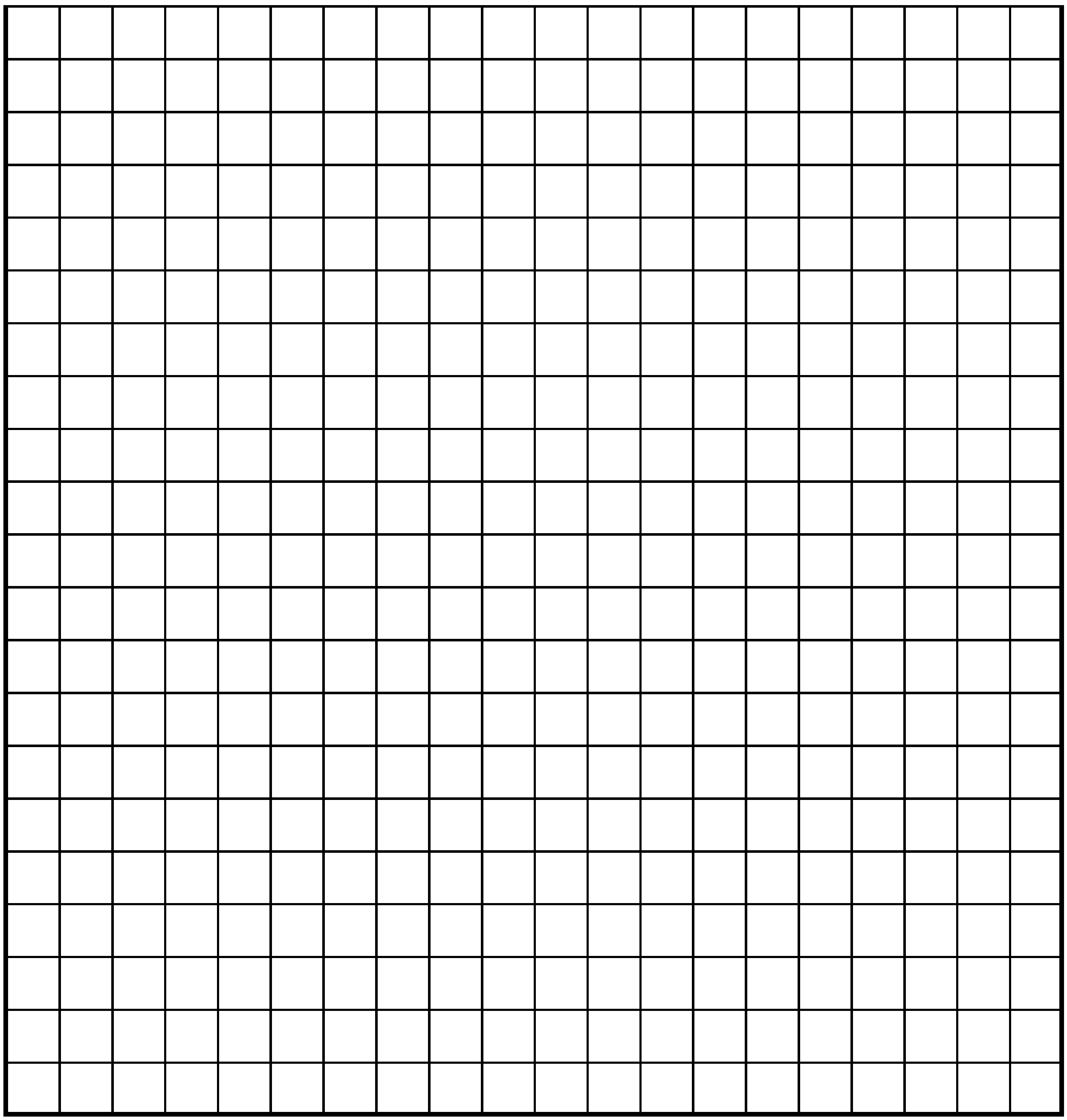

Figure 4. *Extension - Force versus Vertical Deflection*

3. In the Data, Observation, & Analysis section, we asked you to draw a free-body diagram of the mass cantilever system. As an extension, draw a free body diagram of the cantilever in the system with mass added. Include the following: the force of the clamp, the force of gravity (for the hanging and clamped portions), applied force, and normal force.

Assessment

1. Using your equation from question 13 in the Data, Observations & Analysis section, create a generalized formula that relates force, displacement, and the spring constant. Explain your reasoning.

2. What variables must be valid for a system to exhibit Hooke's law?

3. How is the resulting change in the system of force exerted on a cantilever similar to changes that would result from a force exerted on a spring?

4. What would Hooke's law look like if we used a less flexible cantilever? What about a more flexible cantilever? Explain your thinking. Can you think of a general rule for relating a material's physical characteristic to the spring constant?